Page-efficient Encrypted Multi-Maps: New Techniques for Optimal Search Bandwidth

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Abstract

Encrypted multi-maps (EMMs) allow a client to outsource a multi-map to an untrusted server and then later retrieve the values corresponding to a queried label. They are a core building block for various applications such as encrypted cloud storage and searchable encryption. One important metric of EMMs is memory-efficiency: most schemes incur many random memory accesses per search query, leading to larger overhead compared to plaintext queries. Memory-efficient EMMs reduce random accesses but, in most known solutions, this comes at the cost of higher query bandwidth.

This work focuses on EMMs run on SSDs and we construct two page-efficient schemes—one static and one dynamic—both with optimal search bandwidth. Our static scheme achieves $\widetilde{O}(\log N/p)$ page-efficiency and $O(1)$ client storage, where N denotes the size of the EMM and p the SSD's page size. Our dynamic scheme achieves forward and backward privacy with $\widetilde{O}(\log N/p)$ page-efficiency and $O(M)$ client storage, where M denotes the number of labels. Among schemes with optimal server storage, these are the first to combine optimal bandwidth with good page-efficiency, saving up to $O(p)$ and $\widetilde{O}(p\log\log(N/p))$ bandwidth over the state-of-the-art static and dynamic schemes, respectively. Our implementation on real-world data shows strong practical performance.

1 Introduction

An *Encrypted multi-map* (EMM) [15, 28] enables a client to securely outsource a multi-map MM (*i.e.*, a mapping from labels to sets of values) to an untrusted server and later privately query the multi-map for a particular label. We denote by N and M the number of label-value pairs and labels, respectively. Because EMMs are built using fast symmetric-key primitives, they provide a practical and scalable solution for secure cloud storage. For example, EMMs are a key building block of searchable encryption (SSE) (see *e.g.*, [38, 31, 30, 16, 11, 10, 35, 6, 25] and for a survey [23]); they often form the core of the SSE index-based structure. (In this work, we therefore use the notions of SSE and EMM interchangeably when referring to prior works.) Other privacy-preserving applications of EMMs include, encrypted relational databases [27, 12], encrypted graphs [21, 14], private range search [19, 22], and encrypted gun registries [29].

Dynamic EMMs and Leakage. While much research has focused on *static* SSE and EMM schemes, for many such practical applications, it is often desirable for the EMM to be *dynamic* and support updates to the underlying multi-map. The efficiency of both static and dynamic EMMs, however, comes at a cost of some small amount of well-defined information about the queries or underlying dataset being leaked to the server; this information is captured in the form of a leakage function. Security of the EMM ensures that the server only learns limited information captured a leakage function.

An EMM scheme's leakage function is composed of *setup leakage*, *query leakage*, and—in the case of a dynamic scheme—*update leakage*. Setup leakage is the information revealed prior to query execution and comprises the size of the index (or some upper bound on the index size). When a search query is issued, EMM schemes typically leak whether the same query has been made before (the *query pattern*) and the size of the response (the *volume*). For dynamic EMMs, additional security properties are desired due to known file-injection attacks against prior schemes (*e.g.*, [44]). In particular, it is desirable that updates reveal no information about which label was modified, a property known as *forward privacy* [43]. Another important goal, is that no information about the deleted values during query processing is leaked after a deletion, a property called *backward privacy* [9].

Memory-efficiency for EMMs. EMMs are often built by mapping encrypted label-value pairs to a random location in a hash table (*e.g.*, [16, 11, 7]). The locations for a given label ℓ can only be determined by the client, however, the server can recompute these locations given a search token P for ℓ, often derived by a PRF evaluation by the client. The computational overhead of a search query is very low as it relies on efficient symmetric cryptographic operations such as

encryption and PRF evaluations. However, as EMMs operate over large quantities of data, the random access of hash table locations becomes the main bottleneck of EMMs [13]. This behavior is visualized in Fig. 1a. Since only the locations storing an encrypted value v matching label ℓ are accessed, non-memory-efficient EMMs often achieve optimal search bandwidth.

Given the above, Cash and Tessaro [13] introduce the notion of locality (the number of disjunct memory accesses) and read efficiency (the amount of data read beyond the result itself) which are good predictors for the memory-efficiency on HDDs. Later, Bossuat et al. [6] introduce the notion on page efficiency (the number of pages read beyond the minimal requirement) which is tailored to memory-efficiency on SSDs. The area of memory-efficiency for EMMs is an active area of research [10, 34, 2, 20, 3, 18, 4, 6, 35, 36, 37] and many tradeoffs are known. However, *all* known constructions with constant storage efficiency—*i.e.*, encrypted EMMs of size $O(N)$ incur a noticeable bandwidth overhead. We thus ask the following question:

Can we build page-efficient SSE without bandwidth overhead?

1.1 Our Contributions

We answer our research question affirmatively. That is, we build two EMM schemes, S1C and D1C, with constant storage efficiency, $\widetilde{O}(\log N/p)$ page efficiency where p denotes the page size, and optimal search bandwidth (*i.e.*, on search at most $O(n)$ data is communicated for a plaintext result of size n). The schemes have different tradeoffs in terms of client storage and functionality:

- S1C is static and does not require client storage (beyond key material). Its leakage profile matches that of typical EMMs: setup reveals the multi-map size N and search reveals the volume and query pattern.
- D1C is dynamic and has forward and backward privacy. Setup and search leakage are in S1C. In line with other forward-private EMMs with optimal search bandwidth (*e.g.*, [7, 9]), D1C requires $O(M)$ client storage, where M is the number of labels.

On a technical level, we structure the memory access of our EMMs such that the pattern resembles prior page-efficient schemes (see Fig. 1b), however we develop techniques to ensure that we *only* access and return the desired values (see Fig. 1c). We give an overview in Section 1.2 and provide a comparison of our schemes to the state-of-the-art in Table 1.

Instantiation and Evaluation. We implemented our schemes and our evaluation demonstrates excellent performance (see Section 6 for details). We also compare our constructions to state-of-the-art EMMs for concrete data sets [1]. Our analysis shows that we save up to 95x and 72x in bandwidth over static and dynamic page-efficient EMMs, while page efficiency remains comparable (see Appendix A for details).

1.2 Technical Overview

Let us give a brief overview of our techniques. We denote the multi-map by MM and the values matching label ℓ by $\mathsf{MM}[\ell]$.

Techniques for Page-efficient EMMs. Our goal is to construct page-efficient EMMs, *i.e.*, EMMs with efficient memory access on SSDs. To achieve page efficiency $O(E)$, it is sufficient if values associated to ℓ, denoted $\mathsf{MM}[\ell]$, are partitioned into pages $V_1, \ldots, V_n$ of values which are mapped to $n \cdot O(E)$ arbitrary pages on the server, where n denotes the minimal number of pages required to store $\mathsf{MM}[\ell]$. At most one set V_i contains fewer than p values. We refer to such a set as an *incomplete page*, and all other sets as *full pages*. Notably, values within the same set V_i must be stored close together for memory efficiency, while values in other sets V_j for $j \neq i$ can be stored farther apart.

The requirement that values within the same set V_i must be stored in few pages creates correlations between memory locations and labels, which naively leaks information on the multi-map distribution. To tackle this issue, most prior works build on the approach of Asharov et al. [2]. For the i-th set V_i of values matching label ℓ, let P_i denote a search token derived from ℓ and i, *e.g.*, via a PRF. Then, each set V_i is mapped to bins $\mathbf{B} = (B_1, \ldots, B_m)$ via weighted hashing: A (small) subset of bins to store V_i is chosen via a hash function $\mathcal{H}(P_i)$ evaluated on the search token P_i. The weighted hashing variant determines the number of bins accessed and the size of each bin. Notable weighted hashing variants in SSE literature include cuckoo hashing [39, 6], two-choice [42, 2, 35] and one-choice [5, 2]. In one-choice, the set V_i is mapped to a single bin. In two-choice and cuckoo hashing, two possible bins are chosen and V_i is mapped to one of the two, based on the load of the bins (two-choice) or according to an eviction strategy (cuckoo hashing).

In all prior works, the bins B_i contain (at least) p values and for each search query, all possible bins that might contain the values (determined by weighted hashing) are sent by the server to the client. This is visualized in Fig. 1b and is the core reason for the bandwidth overhead in page-efficient EMMs. In particular, as bins are at least of size p, all page-efficient schemes have an overhead $\Omega(p)$ linear in the page size.

Our Idea. Similar to prior work, we employ weighted hashing in our constructions to structure data for good page-efficiency. However, we go one step further and additionally structure the bins. Our core insight is that if B_i is built for efficient retrieval,

[1] Our comparison is analytical: either prior schemes were not implemented or the instantiation did not compile. Therefore, we compare page accesses, storage and bandwidth with prior works asymptotically.

Table 1 – An overview of relevant EMM schemes with optimal server storage efficiency. Static schemes are given in the upper half. We consider dynamic schemes with forward and backward security in the bottom half.

Scheme	Persistent Storage		Search		Update	
	Client	Server	Page Eff.	Bandwidth Overhead	Page Eff.	Bandwidth
PiBas [10]	$O(1)$	$O(N)$	$O(p)$	$O(1)$	-	-
1C [2]	$O(1)$	$O(N)$	$\widetilde{O}(\log N)$	$\widetilde{O}(\log N)$	-	-
2C [2]	$O(1)$	$O(N)$	$\widetilde{O}(\log\log N)$	$\widetilde{O}(\log\log N)$	-	-
Tethys [6]	$O(p\log\lambda)$	$O(N)$	$O(1)$	$O(p)$	-	-
S1C (cf. Section 3)	$O(1)$	$O(N)$	$\widetilde{O}\left(\log\frac{N}{p}\right)$	$O(1)$	-	-
Diana [9]	$O(M)$	$O(N)$	$O(p)$	$O(1)$	$O(1)$	$O(1)$
Hermes [36]	$O(M)$	$O(N)$	$\widetilde{O}\left(\log\log\frac{N}{p}\right)$	$\widetilde{O}\left(p\cdot\log\log\frac{N}{p}\right)$	$\widetilde{O}\left(\log\log\frac{N}{p}\right)$	$\widetilde{O}\left(p\cdot\log\log\frac{N}{p}\right)$
L-SD$_d$[1C] [37]	$O(1)$	$O(N)$	$\widetilde{O}(\log N)$	$\widetilde{O}(\log N)$	$O(\log^2 N)$	$O(\log^2 N)$
L-SD$_d$[2C] [37]	$O(1)$	$O(N)$	$O(\log N)$	$O(\log N)$	$O(\log^3 N)$	$O(\log^3 N)$
D1C (cf. Section 5)	$O(M)$	$O(N)$	$\widetilde{O}\left(\log\frac{N}{p}\right)$	$O(1)$	$\widetilde{O}\left(\log\frac{N}{p}\right)$	$\widetilde{O}\left(p\cdot\log\frac{N}{p}\right)$

N denotes an upper bound on the multi-map size, M an upper bound on the number of labels, and p the page size. The "Bandwidth OH" column under "Search" reports the worst-case factor n_ℓ/n_s, where $n_\ell = |\mathsf{MM}[\ell]|$ is the plaintext response size and n_s the communication during EMM search on label ℓ. The "Bandwidth OH" column under "Update" gives the per-update bandwidth. Note that p accesses per page represent the worst case (cf. PiBas [10], Diana [9]).

we can avoid sending the full bin and instead just retrieve the encrypted values in B_i that match the queried label; values associated to some label ℓ are stored close-together within a few bins B_i *and* only the matching values within B_i are retrieved and sent. This is visualized in Fig. 1c.

While the idea is natural in hindsight, implementing it may introduce additional leakage since the exact memory locations of items could be revealed. It thus requires careful design choices and analysis for a secure and efficient instantiation.

1.2.1 Our Static Scheme

Let us discuss our static EMM S1C based on the above idea. In S1C, the bins B_i are structured as hash tables that allow to retrieve individual values directly. We therefore denote $T_i := B_i$ to stress that T_i is a table. Observe that accessing specific locations in T_i reveals the exact positions of values in the bins. If the bin choice depends on the multi-map's distribution, this compromises security! For example, in weighted cuckoo hashing or weighted two-choice hashing, a value can be mapped to either of two bins, T_i or T_j. Since the multi-map's distribution determines whether a value is inserted into T_i or T_j, we cannot reveal the exact location.

We observe that this issue does not occur for one-choice allocation (1C) since $\mathcal{H}$ maps to exactly one bin T_i. In our security analysis, we prove that we can reveal the exact location of values in B_i without loss of security. We base our construction on Pluto [6], but replace cuckoo hashing with 1C and structure the bins as discussed above. Incomplete pages are allocated using 1C and each bin is structured as hash table, whereas full pages are stored in simple hash tables. This avoids the bandwidth overhead, while preserving a page efficiency of $\widetilde{O}(\log N/p)$.

1.2.2 Our Dynamic Scheme

We show that our idea also applies to the dynamic setting. In this setting, page-efficiency is challenging since forward privacy requires that no information is leaked about which label is updated. Simultaneously, values associated to the same label must be stored close in storage (*i.e.*, label and storage locations are correlated). There are two solutions to this issue that preserve forward and backward privacy.

1. The first approach [36] employs an EMM with *dummy updates* to achieve $\widetilde{O}(\log\log N/p)$ page efficiency for search and updates, and a client storage of $O(M)$.

2. The second approach [37] is based on hierarchical SSE [17]. Updates are implemented based on oblivious sorts and oblivious compaction. As opposed to [36], the schemes in [37] have (almost) no client storage requirement. However, [37] has page efficiency and bandwidth $O(\log N)$ due to the fact that hierarchical structure and updates rely on generic oblivious tools.

Our dynamic constructions build on the Hermes framework. Extending our techniques to [37] to optimize search bandwidth is an interesting direction, though achieving optimal bandwidth may be difficult due to its hierarchical structure

Extending the Hermes Framework [36]. For convenience, we partition update queries into epochs, or sequences of M

consecutive updates. Hermes buffers each update at the client, and later pushes them to the server via an oblivious schedule (see Fig. 2). During a search, values not yet pushed are retrieved from the client buffer. Client storage scales linearly with the number of labels, as is typical for efficient forward-private SSE.

Our scheme also inherits $O(M)$ client storage; we note that lower bounds indicate that this is hard to avoid [8] and known techniques for constant client storage incur at least a logarithmic overhead in search bandwidth, *e.g.*, hierarchical EMM structure [17, 37] or the use of ORAM [24]. Below, we recap the Hermes framework [36] and discuss our modifications to obtain optimal search bandwidth. We discuss an amortized variant, where each M updates up to $O(M)$ pages are read. However, it is possible to deamortize the construction and we refer Section 5 for details. The page size p and upper bound on MM's size N and number of labels M determines whether the multi-map is sparse (*i.e.*, the number of values per label on average is less than the page size, or $N < pM$) or dense (*i.e.*, $N \geq pM$). Hermes, and therefore our variant, proceeds in different ways, depending on whether the multi-map falls into the sparse or dense regime. This is explained below.

(1) Sparse Regime: In the sparse regime, [36] builds on a static EMM. Minaud and Reichle [36] observe that for every M updates (*i.e.*, every epoch), it is possible to read the entire encrypted multi-map EMM with *amortized* $O(1)$ update page efficiency, assuming that EMM is of size $O(N)$.

This is easy to see as $N < pM$ in the sparse regime, thus the whole multi-map can be stored in $O(M)$ pages. This observation is leveraged by buffering M values on the client, then downloading the entire encrypted multi-map from the server to insert the buffered values on the client locally (*e.g.*, by reencrypting the multi-map). Note that this rebuild process is *forward private*, *i.e.*, leaks no additional information, and can be deamortized.

In the sparse regime, Hermes builds on LayeredSSE [35], a static EMM with $\widetilde{O}(\log\log N/p)$ page efficiency and search bandwidth. Hermes' performance is inherited from LayeredSSE. On the other hand, we base our dynamic construction D1C on our static EMM S1C and build a deamortized update mechanism (cf. Section 5 for details). As in [36], D1C inherits the performance characteristics from S1C, including its good page efficiency and $O(1)$ search bandwidth.

(2) Dense Regime: In this regime, the authors employ two structures that store either incomplete or full pages of values. The first is a collection of bins $\mathbf{B} = (B_1, \ldots, B_M)$. Each bin B_i stores an incomplete page associated to the i-th label ℓ (*i.e.*, up to $p-1$ values). This structure is only storage efficient in the dense regime. The second structure is an EMM scheme Σ that stores full pages and supports dummy updates, which are indistinguishable from real updates and incur no additional storage overhead.

In the dense regime, all M bins $\mathbf{B}$ can be read with $O(1)$ page efficiency each epoch. Whenever a bin is full, it is emptied and its content is moved to the scheme Σ that stores full pages. To ensure forward privacy, the induced update operations on Σ are made independent of whether the bin was full or not. Further, in M updates, at most $\lceil M/p \rceil$ full pages can be added in total and at most M bins can be filled; therefore $2M$ update or dummy updates suffice per M updates. Again, the scheme can be deamortized. Efficiency is inherited from Σ and the bins: if Σ reads E distinct locations each search or update, then the scheme has $O(E)$ page efficiency (as each access retrieves or stores full pages). A bin access is naturally page-efficient.

To instantiate the framework, we build a novel EMM Dummy1C that supports dummy updates based on one-choice. Dummy1C has optimal search bandwidth and reads $E = O(1)$ memory locations on search. (For more details on Dummy1C, we refer to Section 4. We remark that our aforementioned deamortization technique in the other regime also relies on Dummy1C.) Furthermore, we structure the bins $\mathbf{B}$ as tables $\mathbf{T} = (T_1, \ldots, T_M)$. As in our static scheme S1C, this allows us to avoid bandwidth overhead on search.

In total, we obtain optimal search bandwidth with $\widetilde{O}(\log N/p)$ page efficiency in both regimes. Also, we have optimal storage efficiency (*i.e.*, the encrypted multi-map is of size $O(N)$) and update page efficiency and communication is $\widetilde{O}(\log N/p)$ (in the worst-case).

2 Preliminaries

Notation. We follow the notation of [36]. We denote the security parameter as λ. For a probability distribution X, we write $x \leftarrow X$ for sampling a value x from X. For a set $\mathcal{X}$, we write $x \leftarrow \mathcal{X}$ for sampling uniformly at random from $\mathcal{X}$. A function $f(\lambda)$ is *negligible* in λ if it is $O(\lambda^{-c})$ for every $c \in \mathbb{N}$, and we write $f = \mathsf{negl}(\lambda)$ for short. Similarly, f is overwhelming in λ if $f = 1 - \mathsf{negl}(\lambda)$.

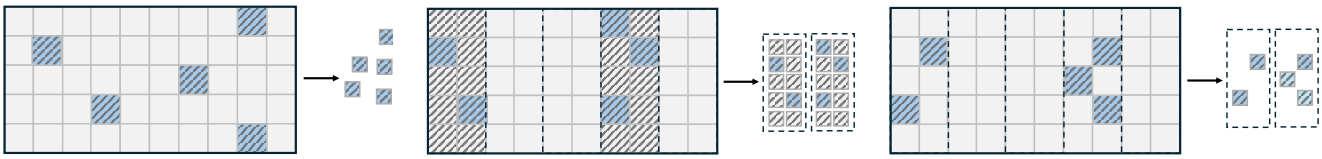
For a protocol $\mathsf{prot} = (\mathsf{prot}_A, \mathsf{prot}_B)$ between two parties A and B, we denote by $\mathsf{prot}_\mathsf{A}(\mathsf{opin}_A) \longleftrightarrow \mathsf{prot}_\mathsf{B}(\mathsf{opin}_B)$ an execution of protocol between A and B with input opin_A and opin_B, respectively. Sometimes, we write $\mathsf{prot}(\mathsf{opin}_A; \mathsf{opin}_B)$ if both executing parties can be inferred from the context.

We primarily use hash tables and bins as data structures. For readability, we assume that tables mapping to integers are initialized with 0 entries and that bins are padded to a fixed capacity (if it can be inferred from the context).

2.1 Data Structures

In this work, we employ hash tables and bins frequently.

Hash Tables. A table T that is initialized for size N allows to store up to N key-value pairs (k, v). We write $v \leftarrow T[k]$ to retrieve the value v associated to k and we write $T[k] \leftarrow v$ to store v under key k. In general, when parties communicate T, we assume the party holding T sends (up to) N key-value



(a) A non-page-efficient EMM with random access and optimal bandwidth.

(b) A page-efficient EMM with with bandwidth linear in the page size.

(c) Our solution for a page-efficient EMM with optimal bandwidth.

Figure 1 – An overview of how EMM schemes differ with respect to memory access and bandwidth costs. Blue squares denote the addresses of the queried for records and shaded squares denote the addresses of records that are returned by the server to the client. Prior work falls into categories (a) and (b), and in this work, we present the first scheme that belongs to category (c).

pairs $(k_i, v_i)_{i\in[N]}$. The receiving party can structure T in such a way that retrieval and updates are efficient.

In Appendix D.2, we introduce a hash table variant that does not leak the keys k_i except on access of position k_i. We employ different notation for such tables.

Bins. A bin B of capacity cap is a segment of data of size cap that contains arbitrary information. We assume bins are always padded to full capacity cap.

2.2 Cryptographic Primitives and Allocation

We employ pseudorandom functions (PRFs) and (symmetric) encryption schemes, and we prove security in the random oracle model. We give a brief overview of these concepts below, and refer to [32] for formal definitions.

PRF. A pseudorandom function (PRF) is a function F that maps keys $k \in \{0,1\}^\ell$ and inputs in $x \in \{0,1\}^n$ to values in $y \in \{0,1\}^m$. We write $y = F_k(x) := F(k,x)$ for short. We say that F is secure if for some $k \leftarrow \{0,1\}^\ell$, it is difficult to distinguish between F_k and a truly random function.

Encryption. An encryption scheme is a tuple of PPT algorithms $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$, where KeyGen outputs a secret key $\mathsf{K}_{\mathsf{Enc}}$ that allows to encrypt and decrypt messages, *i.e.*, $c \leftarrow \mathsf{Enc}(\mathsf{K}_{\mathsf{Enc}}, m)$ and $m = \mathsf{Dec}(\mathsf{K}_{\mathsf{Enc}}, c)$. We write $c = \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(m) := \mathsf{Enc}(\mathsf{K}_{\mathsf{Enc}}, m)$ for short. We say that Enc is IND-CPA secure, if it is difficult to distinguish between encryptions of m_0 and m_1 of identical length.

Random Oracle Model. In the random oracle model, the adversary has oracle access to some hash function(s) $\mathcal{H}_1, \ldots, \mathcal{H}_n$. In the security game, these are modeled as programmable random functions.

2.3 One-choice Allocation

In one-choice allocation ($\mathsf{1C}$), there are n balls of weight 1 thrown into m bins such that each ball is allocated to a single, random bin. In weighted $\mathsf{1C}$, each ball is associated with a weight $0 \le w \le 1$ and their total weight is at most n. For both variants, there are (either explicit or implicit) logarithmic upper bounds in literature, *i.e.*, the most loaded bin has weight at most $\widetilde{O}(\log n)$ except with negligible probability. We will employ $\mathsf{1C}$ allocation to place multi-map values into bins: A list L of up to p values is placed into a random bin (chosen by a hash function) and is interpreted as ball of weight L/p. We refer to Appendix B for more details.

2.4 Encrypted Multi-maps

A multi-map $\mathsf{MM} = \{(\ell_i, (v_1, \ldots, v_{n_i}))\}_{i=1}^{L}$ is a set of label-value pairs with L total labels. We denote by $\mathsf{MM}[\ell_i] = (v_1, \ldots, v_{n_i})$ the list of values matching label ℓ_i. As is common in literature, we assume that there is an upper bound M on the total number of labels $L \le M$ and an upper bound N on the size of the multi-map $\sum_{i=1}^{L} |\mathsf{MM}[\ell_i]| \le N$. We denote the label space as $\mathcal{L}_{\mathsf{MM}}$ and the value space as $\mathcal{V}_{\mathsf{MM}}$. We omit the subscript MM if clear by context. We write $|\mathcal{V}|_b$ and $|\mathcal{L}|_b$ for an upper bound on the bit-size of values and labels, respectively. Further, we denote by p the page size (which is treated as variable independent of the size of the database).

We recall the definition of a dynamic EMM scheme (DEMM) and related notions following [15, 28, 36, 1].

Definition 1. *A dynamic EMM scheme* Σ *is a tuple of 2 algorithms and 2 protocols* $(\mathsf{KeyGen}, \mathsf{Setup}, \mathsf{Search}, \mathsf{Update})$*:*

- $\Sigma.\mathsf{KeyGen}(1^\lambda)$*: a PPT algorithm that takes as input the security parameter* λ *and outputs client secret key* K*.*
- $\Sigma.\mathsf{Setup}(\mathsf{K}, N, M, \mathsf{MM})$*: a PPT algorithm that takes as input client secret key* K*, upper bounds* N *on the multi-map size and* M *on the number of values, and a multi-map* MM*. It outputs encrypted multi-map* EMM *and client state* st*.*
- $\Sigma.\mathsf{Search}(\mathsf{K}, \ell, \mathsf{st}; \mathsf{EMM})$*: a protocol executed by the client and server. The client receives as input the secret key* K*, label* ℓ *and state* st*. The server receives as input the encrypted multi-map* EMM*. It outputs some data* d *and updated state* st' *for the client; it outputs updated encrypted multi-map* EMM' *for the server.*
- $\Sigma.\mathsf{Update}(\mathsf{K}, \ell, v, \mathsf{op}, \mathsf{st}; \mathsf{EMM})$*: a protocol executed by the client and server. The client receives as input the secret key* K*, a label-value pair* (ℓ, v)*, an operation* $\mathsf{op} \in \{\mathsf{del}, \mathsf{add}\}$ *and state* st*. The server receives as input the encrypted multi-map* EMM*. Outputs updated state* st' *for the client. Outputs updated encrypted multi-map* EMM' *for the server.*

Conventions. We use the following conventions.

- If the client state st is clear from context, we omit st and assume that it is stored and updated by the client.
- We sometimes omit the parameters MM and M in Setup if not they are not required (*e.g.*, if the scheme is forward-private and setup can be done interactively via Update with no additional leakage). If MM is omitted in Setup, we assume that $|\mathcal{V}|_b$ and $|\mathcal{L}|_b$ are still passed as input implicitly.
- All EMM schemes that support an "add" operation can be extended to support deletes with the generic framework of [7, 9] while maintaining efficiency (up to a factor 2). For readability, we present our constructions without deletions and assume that $\mathsf{op} = \mathsf{add}$ in Update.

2.4.1 Correctness and Security

Correctness. Correctness ensures that when a label ℓ is queried for, the client obtains $\mathsf{MM}[\ell]$.

Definition 2 (Correctness)**.** *A* EMM *scheme* Σ *is correct if for all databases* DB *and* $M, N \in \mathbb{N}$*, keys* $\mathsf{K} \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$*,* $\mathsf{EMM} \leftarrow \Sigma.\mathsf{Setup}(\mathsf{K}, N, M, \mathsf{MM})$ *and sequences of search, add or delete queries* S*, the search protocol returns the correct result for all queries of the sequence if the size of the database remains at most* N *and at most* M *keywords are used.*

Security. Semantic security ensures that the server learns information quantified by a leakage function. Let $\mathcal{L} = (\mathcal{L}_{\mathsf{Stp}}, \mathcal{L}_{\mathsf{Srch}}, \mathcal{L}_{\mathsf{Updt}})$ be a leakage function, where $\mathcal{L}_{\mathsf{Stp}}$ denotes setup leakage, $\mathcal{L}_{\mathsf{Srch}}$ the search leakage, and $\mathcal{L}_{\mathsf{Updt}}$ the update leakage. The adversary and challenger interact in EMM_{REAL} or $\text{EMM}_{\text{IDEAL}}$ as defined below. First, the honest-but-curious adversary chooses a multi-mapp MM. In EMM_{REAL}, the encrypted multi-map EMM is generated by $\mathsf{Setup}(1^\lambda, N, M, \mathsf{MM}; 1^\lambda)$, whereas in $\text{EMM}_{\text{IDEAL}}$ the encrypted multi-map is simulated by a (stateful) simulator Sim on input $\mathcal{L}_{\mathsf{Stp}}(\mathsf{MM}, N, M)$. After receiving EMM, the adversary adaptively issues search and update queries. All queries are answered via the corresponding client protocol in EMM_{REAL}. In $\text{EMM}_{\text{IDEAL}}$, the search queries on label ℓ are simulated by Sim on input $\mathcal{L}_{\mathsf{Srch}}(\ell)$ and update queries for operation op, keyword ℓ and value v are simulated by Sim on input $\mathcal{L}_{\mathsf{Updt}}(\mathsf{op}, \ell, v)$. The adversary outputs a bit b.

Definition 3 (Semantic Security)**.** *Let* Σ *be an* EMM *scheme and* $\mathcal{L} = (\mathcal{L}_{\mathsf{Stp}}, \mathcal{L}_{\mathsf{Srch}}, \mathcal{L}_{\mathsf{Updt}})$ *a leakage function. Scheme* Σ *is* $\mathcal{L}$*-adaptively secure if for all honest-but-curious PPT adversaries* $\mathcal{A}$*, there exists a PPT simulator* Sim *such that*

$$|\Pr[\text{EMM}_{\text{REAL}}{}_{\Sigma,\mathcal{A}}(\lambda) = 1] - \Pr[\text{EMM}_{\text{IDEAL}}{}_{\Sigma,\mathsf{Sim},\mathcal{L},\mathcal{A}}(\lambda) = 1]| = \mathsf{negl}(\lambda).$$

Semantic security guarantees that each query reveals no information to the server, beyond the leakage of the query.

Leakage Patterns. We use standard notions of search pattern sp, update pattern up, query pattern qp and history Hist:

(1) The search pattern $\mathsf{sp}(\ell)$ for a label ℓ comprises of the values corresponding to previous search queries. (2) The update pattern $\mathsf{up}(\ell)$ for a label ℓ comprises of the values corresponding to previous update queries. (3) The query pattern $\mathsf{qp}(\ell) = (\mathsf{sp}, \mathsf{up})$ for a label ℓ is the indices of previous search or update queries for a label ℓ. (4) The history $\mathsf{Hist}(\ell)$ comprises of the list of values matching label ℓ that was inserted during setup and the history of updates on label ℓ, that is each deleted and inserted value. We can retrieve the number t of inserted values and the number d of deleted values from $\mathsf{Hist}(\ell)$ for each label.

Forward and Backward Privacy. Forward privacy was introduced in [43]. Here, we target the stronger variant introduced in [26], *i.e.*, we ask that updates leak no information.

Definition 4. *A DEMM scheme is* forward-private *if it is secure with respect to a leakage function* $\mathcal{L} = (\mathcal{L}_{\mathsf{Stp}}, \mathcal{L}_{\mathsf{Srch}}, \mathcal{L}_{\mathsf{Updt}})$ *such that* $\mathcal{L}_{\mathsf{Updt}} = \bot$.

Backward privacy [9] restricts the leakage incurred by deletions. We consider type-II backward privacy which requires that search queries leak at most the values currently matching label ℓ, when they were inserted, and when all the updates on ℓ happened (but not their content). We refer to [9] for formal definitions. By convention, we do not supply an explicit deletion operation (cf. Section 2.4) and employ the framework by [7] to augment our schemes to support deletes, combined with the techniques in [9, Section 4.3] to obtain type-II backward privacy. Note that while generic, this transformation is efficient and forms the basis of most modern and efficient type-II backward-private schemes. Roughly, deletes are stored in an additional encrypted multi-map $\mathsf{EMM}_{\mathsf{del}}$, that is, to delete label-value pair (ℓ, v), the pair (ℓ, v) is added to $\mathsf{EMM}_{\mathsf{del}}$. On search, the client searches label ℓ on both EMMs and obtains two sets of values $\mathcal{V}$ and $\mathcal{V}_{\mathsf{del}}$. For backward-privacy, the client sets $\mathcal{R} = \mathcal{V} \setminus \mathcal{V}_{\mathsf{del}}$ as result.

2.4.2 EMM Supporting Dummy Updates.

The notion of EMM supporting dummy updates was introduced in [36]. Intuitively, such an SSE scheme allows to make *dummy* updates that are indistinguishable from real updates, but do not change the content of the database. This procedure is denoted by $\mathsf{DummyUpdate}$, which takes the client secret key K and client state st as input. Also, Setup receives an additional parameter D, an upper bound on the number of possible dummy updates. We refer to Appendix D.1 for a formal definition.

2.4.3 Efficiency Measures.

We now ecall the notions of storage efficiency [13] and page efficiency [6]. Below, we set $\mathsf{K} \leftarrow \mathsf{KeyGen}(1^\lambda)$ and $\mathsf{EMM} \leftarrow \mathsf{Setup}(\mathsf{K}, N, M, \mathsf{MM})$, and $S = (\mathsf{op}_i, \mathsf{opin}_i)_{i=1}^s$ is a sequence of search and update queries, where $\mathsf{op}_i \in \{\mathsf{add}, \mathsf{del}, \mathsf{srch}\}$ is an operation and $\mathsf{opin}_i = (\mathsf{op}_i, \ell_i, v_i, \mathsf{st}_i; \mathsf{EMM}_i)$ its input. After executing all previous operations op_j for $j \le i$, st_i is the client state, EMM_i the encrypted multi-map, and MM_i the plaintext multi-map. We assume that the total number of identifiers (resp. values) never exceeds N (resp. M). Also, if $\mathsf{op}_i = \mathsf{srch}$, the query is a search query and a value is not provided.

Definition 5 (Storage Efficiency)**.** *An EMM scheme has storage efficiency E if for any $\lambda, N \in \mathbb{N}$, multimap* MM*, sequence S, and any i, $|\mathsf{EMM}_i| \le E \cdot |\mathsf{MM}_i|$.*

Definition 6 (Page Pattern)**.** *Regard server-side storage as an array of* pages*, containing the encrypted multi-map* EMM*. When processing search query $\mathsf{Search}(\mathsf{K}, \ell_i, \mathsf{st}_i; \mathsf{EMM}_i)$ or update query $\mathsf{Update}(\mathsf{K}, \ell_i, v_i, \mathsf{op}_i, \mathsf{st}_i; \mathsf{EMM}_i)$, the server accesses a number of pages $p_1, \ldots, p_{h'}$. We call these pages the* page pattern*, denoted by $\mathsf{PgPat}(\mathsf{op}_i, \mathsf{opin}_i)$.*

Definition 7 (Page Efficiency)**.** *An EMM scheme has page efficiency P if for any $\lambda, N \in \mathbb{N}$, multi-map* MM*, sequence S, and any i, $|\mathsf{PgPat}(\mathsf{op}_i, \mathsf{opin}_i)| \le P \cdot X$, where X is the number of pages accessed to answer the query in the plaintext* MM*.*

Definition 8 (Communication Efficiency)**.** *An EMM scheme has communication efficiency C if for any $\lambda, N \in \mathbb{N}$, multi-map* MM*, sequence S, and any i, we have $R \le C \cdot X$, where R is the total client-server communication when operation op_i with input opin_i is performed on EMM, where X is the size of the answer to the query in plaintext multi-map* MM*.*

3 Our Static EMM

To motivate the approach for how we structure the bins in our dynamic scheme in Section 4, we first describe a "simpler" static scheme with a similar approach. Our static scheme, S1C, is the first EMM scheme—to our knowledge—to achieve good memory efficiency, whilst also ensuring optimal communication and storage efficiency. It achieves $\widetilde{O}(\log N/p)$ page efficiency and $O(1)$ communication and storage efficiency.

Scheme Overview. At a high level, this scheme is like the *Pluto* scheme [6], except that the incomplete bins are replaced with multiple cuckoo tables together with a one choice hash function. More specifically, in our static scheme, the client initializes two tables, T_{full} and T_{len}, of lengths $\lceil N/p \rceil$ and N, respectively. Let MM be the plaintext multimap that we wish to securely outsource. For each label in MM, the client partitions the corresponding values into pages, encrypts each of the full pages, and stores the encrypted pages in T_{full}. The table T_{len} is then used to store the (blinded) number of values associated with each label. Since not all value sets are multiples of p, we must find a way to handle the overflow values, *i.e.*, those in an incomplete page, for each label. To store these additional values, the client initializes another $m = \lceil N/(p \cdot \delta(\lambda) \cdot \log(N)) \rceil$ empty tables, $T_{\mathsf{val},0}, \ldots, T_{\mathsf{val},m-1}$, encrypts the values, and inserts each encrypted value into these tables using one-choice.

To query for a label, the client generates a query-specific token with which the server can use to retrieve and unblind the number of associated values from T_{len}. The token can futher be used to retrieve the corresponding full pages of values from T_{full} and any additional values that were inserted into the tables $T_{\mathsf{val},\gamma}$ for $\gamma \in [0, m-1]$.

The pseudocode is given in Algorithm 1. We refer to Appendix C.1 for a detailed description. Note that $\mathcal{H}_P$ is a hash function that maps to $\{0,1\}^{2\lambda}$.

Remark. For clarity, we instantiate our static scheme concretely. We note, however, that T_{full} can instead be computed by defining a plaintext multimap $\mathsf{MM}_{\mathsf{full}}$ that maps each label $\ell_i \in \mathsf{MM}$ to the set of corresponding complete pages and then encrypting $\mathsf{MM}_{\mathsf{full}}$ using any static EMM scheme. Since $\mathsf{MM}_{\mathsf{full}}$ stores full pages, this black-box EMM scheme does not itself need to be page efficient. This abstraction will later be useful for understanding our dynamic scheme in Section 4, as we make similar black-box use of a specific type of EMM.

Efficiency. It is easy to see that storage efficiency and communication efficiency is $O(1)$. Page efficiency is $\widetilde{O}(\log N/p) + O(1)$: each search query for a label ℓ with cnt corresponding values, at most $p-1$ distinct positions in $T_{\mathsf{val},\gamma}$ of size $\mathsf{cap} = \widetilde{O}(p \log N/p)$, and at most $\mathsf{pgcnt} = \lfloor \mathsf{cnt}/p \rfloor$ full pages are accessed in T_{full}.

Correctness. Correctness is straightforward. By construction, the full pages are stored in T_{full} at location $P_{i,j}$ and the incomplete pages are stored in the table $T_{\mathsf{val},\gamma}$, where $\gamma = \mathcal{H}_{\mathsf{1C}}(P_{i,\mathsf{pgcnt}})$. The properties of 1C (cf. lemma 9) ensure that no table $T_{\mathsf{val},\gamma}$ overflows.

Security. Intuitively, the setup leakage is the multi-map size N and search leakage is the search pattern sp and volume cnt. We formalize this intuition and give a proof in Appendix E.1. Our analysis relies on the properties of 1C.

4 Our EMM with Dummy Updates

Our dynamic, page-efficient EMM scheme (Section 5), makes black-box use of a dynamic (not necessariy page-efficient) EMM scheme that supports *dummy updates*. Dummy updates were introduced in [36] in order to simulataneously realize forward privacy and I/O efficiency. A dummy update allows the client to issue a query that is indistinguishable from a real update from the server's point of view, without requiring additional storage overhead.

In this section, we present our EMM scheme with dummy updates, Dummy1C, which we use to build our dynamic

Algorithm 1 S1C

S1C.KeyGen(1^λ)

Sample $\mathsf{K}_{\mathsf{PRF}} \leftarrow \{0,1\}^\lambda$ and encryption key $\mathsf{K}_{\mathsf{Enc}}$
return $\mathsf{K} = (\mathsf{K}_{\mathsf{PRF}}, \mathsf{K}_{\mathsf{Enc}})$

S1C.Search(K, ℓ_i; EMM)

Client:

Set $(K_i, \mu_i, P_i') \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\ell_i)$
send (K_i, μ_i, P_i')

Server:

Initialize empty set R
Set $\mathsf{cnt}_i \leftarrow T_{\mathsf{len}}[K_i] \oplus \mu_i$
Set $\mathsf{pgcnt}_i \leftarrow \lfloor \mathsf{cnt}_i / p \rfloor$
for all $1 \le j \le \mathsf{pgcnt}_i$ **do**
    Set $P_{i,j} \leftarrow \mathcal{H}_P(P_i', j)$
    Add $T_{\mathsf{full}}[P_{i,j}]$ to R
Set $P_{i,\mathsf{pgcnt}_i} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i + 1)$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_{i,\mathsf{pgcnt}_i+1})$
for all $j \in [\mathsf{cnt}_i \bmod p]$ **do**
    Set $P_{i,\mathsf{pgcnt}_i+1,j} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i + 1, j)$
    Add $T_{\mathsf{val},\gamma}[P_{i,\mathsf{pgcnt}_i+1,j}]$ to R
send R

Client:

Decrypt R to retrieve $\mathsf{MM}(w)$

S1C.Setup($1^\lambda, \mathsf{K}, N, M, \mathsf{MM}$)

Initialize empty table T_{len} with N entries, each of size $\lceil \log N \rceil$
Initialize empty table T_{full} with $\lceil N/p \rceil$ entries, each of size $p \cdot |\mathcal{V}|_b$
Set $\mathsf{cap} \leftarrow O(p\delta(\lambda)\log(N/p))$ and $m \leftarrow \left\lceil \frac{N}{p \cdot \delta(\lambda) \cdot \log(N/p)} \right\rceil$
for all $\gamma \in \{0, \ldots, m-1\}$ **do**
    Initialize empty table $T_{\mathsf{val},\gamma}$ with cap entires, each of size $|\mathcal{V}|_b$
for all labels ℓ_i **do**
    Set $V_i \leftarrow \mathsf{MM}[\ell_i]$ and $\mathsf{cnt}_i \leftarrow |V_i|$
    Set $\mathsf{pgcnt}_i \leftarrow \lfloor \mathsf{cnt}_i / p \rfloor$
    Set $(K_i, \mu_i, P_i') \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\ell_i)$
    Set $T_{\mathsf{len}}[K_i] \leftarrow \mu_i \oplus \mathsf{cnt}_i$
    Partition V_i into $\mathsf{pgcnt}_i + 1$ lists $V_{i,j}$ such that $|V_{i,j}| = p$ for $j \le \mathsf{pgcnt}_i$, and $0 \le |V_{i,\mathsf{pgcnt}_i}| < p$
    for all $1 \le j \le \lfloor \mathsf{cnt}_i / p \rfloor$ **do**
        Set $P_{i,j} \leftarrow \mathcal{H}_P(P_i', j)$
        Set $T_{\mathsf{full}}[P_{i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j})$
    Set $P_{i,\mathsf{pgcnt}_i} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i + 1)$
    Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_{i,\mathsf{pgcnt}_i+1})$
    Set $j \leftarrow 1$
    for all $\mathsf{v} \in M_{i,\mathsf{pgcnt}_i+1}$ **do**
        Set $P_{i,\mathsf{pgcnt}_i+1,j} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i + 1, j)$
        Set $T_{\mathsf{val},\gamma}[P_{i,\mathsf{pgcnt}_i+1,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(\mathsf{v})$
        Set $j \leftarrow j + 1$
Fill T_{len} and T_{full} with random entries
send $\mathsf{EMM} = (T_{\mathsf{len}}, T_{\mathsf{full}}, (T_{\mathsf{val},0}, \ldots, T_{\mathsf{val},\mathsf{m}-1}))$

EMM (Section 5). This scheme supports a $\mathsf{poly}(\lambda)$ number of dummy updates with $\widetilde{O}(\log N)$ communication for updates and $O(1)$ search communication. Notably, Dummy1C gives a different tradeoff to the Dummy(Σ) scheme of Minaud and Reichle [36] which has $\widetilde{O}(\log\log N)$ communication for both updates and searches. We give a brief overview of Dummy(Σ) [36] below.

At a high level, Dummy(Σ) combines a forward-private EMM scheme Σ with two-choice (2C) allocation.[2] Σ is required to be such that each value is stored in a hash table T at a concrete location determined by the scheme. For a real update, instead of inserting v into T at location P (as in Σ), v is placed in the least loaded of two bins B_α, B_β, where $\alpha, \beta \leftarrow \mathcal{H}(P)$. For a dummy update, two random bins are accessed. In both cases, the bins are reencrypted so the server cannot distinguish whether an insertion occurred. During search, the client computes positions P_i via Σ.Search and retrieves the corresponding bins $B_{\alpha_i}, B_{\beta_i}$ for $\alpha_i, \beta_i \leftarrow \mathcal{H}(P_i)$.

We further observe that values can be retrieved directly from bins to reduce communication. To achieve this in Dummy1C, each bin is implemented as a hash table supporting $O(1)$ read access. With 2Callocation, each value could lie in one of two bins, forcing the client to access both. Since revealing the correct bin leaks information (see Appendix D.3.2), we replace 2Cwith 1C, so each value maps to a single bin. We note that our security proof requires special care in handling hash table keys.

The forward security of Σ ensures forward security of Dummy(Σ). Furthermore, because the values are stored in encrypted bins, the server cannot distinguish between real and dummy updates since—in both cases—two (seemingly random) bins are retrieved and reencrypted. The bins only need to be scaled for N values, since dummy updates do not change the content of the bins. This approach has a multiplicative communication blowup of $\widetilde{O}(\log\log N)$ for searches and updates due to the use of 2C allocation.

Because Dummy1C relies on hash tables, care is needed to hide the distinction between real and dummy updates. In standard hash tables, keys are stored in the clear; while these are only hash values, real updates necessarily modify a key whereas dummy updates cannot, which leaks information.

We resolve this by introducing new data structure, a *keyless cuckoo hash table*,which is based on cuckoo-hashing, but modifies the table so that the hash keys are not stored with the items. During lookup, both candidate items are returned, and the client determines which is correct. We describe standard cuckoo hashing and our modified construction in Appendix D.2.

Dummy1C has $O(1)$ and $\widetilde{O}(\log N)$ search and update com-

[2] In 2C, a ball is placed in the least loaded of two bins chosen uniformly at random. After n balls are allocated to $m = N/\widetilde{O}(\log\log N)$ bins, the heaviest bin contains at most $\widetilde{O}(\log\log n)$ balls.

munication efficiency, respectively. A detailed description of the construction and its security can be found in Appendix D and the pseudocode in Algorithm 5.

5 Our Dynamic and Forward-private EMM

We now combine our techniques from Sections 3 and 4 with the Hermes framework [36] to obtain a dynamic EMM scheme with constant search communication and good page efficiency. We refer to Section 1.2 for a recap and assume some familiarity with the framework.

Similar to Hermes, we employ a bin structure, $(T_i)_{i\in[m]}$, to store incomplete pages and an EMM, Σ, which supports dummy updates to store full pages. We instantiate Σ using Dummy1C—our construction from Section 4—and structure each bin as a hash table for efficient access.

The number of tables and their capacity depends on the regime and require $N < pM$ and $N \geq pM$ for the *sparse* and *dense* regimes, respectively. It is convenient to partition updates into epochs, that is, the k-th epoch comprises all updates and searches that are performed between the $((k-1)\cdot M+1)$-th and $(k\cdot M)$-th update.

Sparse Regime: This regime employs one-choice allocation (1C). Each bin is of size $\mathsf{cap} = p\cdot\widetilde{O}(\log N/p)$ and there are $m = O(N/\mathsf{cap})$ bins in total. Each value associated to label ℓ is stored in bin T_i, where $i = \mathcal{H}_{\mathsf{1C}}(P_\ell)$ and P_ℓ is a search token derived from a PRF evaluated on ℓ. When p or more values associated to ℓ are stored in T_i, then a full page of values associated to ℓ is moved to Σ.

Dense Regime: In this regime, each bin is of capacity $\mathsf{cap} = p$ and there are $m = M$ bins in total. A value v associated to label ℓ is stored in the bin $T_{f(\ell)}$, where the bin index is chosen via an injective choice function f. As before, if table T_i is full, then the full page is moved to Σ.

Updates are pushed to the server using a client buffer mechanism, similar to in Hermes [36]. We depict the update schedule for the dense regime in Fig. 2. We stress that, while the schedule looks rather complex, all operations are efficient and comprise of page-efficient memory accesses, local restructuring of data on the client, or rebuilding small bins T_i.

Towards Constant Communication. Since Dummy1C (Section 4) has $O(1)$ search communication, invoking Dummy1C.Search has constant overhead. Naively, if we were to download each bin T_i as in [36], then communication would be $\Theta(\mathsf{cap}) = \Omega(p)$ in the worst case. We thus structure each bin as a hash table, as we did in our static construction (Section 3), to achieve constant communication.

A subtle but important detail is that we need to *rekey* the hash table whenever a bin T_i is updated: The hash table keys leak whether a full page is written to Σ and removed from T_i. It is tempting to employ our keyless hash table from Appendix D.2, but the keys would be revealed on search. As values are removed from the table, the distribution of the seeds for $\mathcal{H}_{\mathsf{CT}}$ may leak whether a previously searched value was removed from T_i: removed keys might not constitute a valid cuckoo assignment. Instead, we set use fresh keys each epoch. When a table T_i is rebuilt in the j-th epoch, then j is employed to derive the keys via a PRF. With this modification, the keys are freshly distributed after the table $_i$ is rebuilt each epoch, independent of whether values were written to or removed from T_i. Since the update schedule of bin T_i is deterministic, the client can derive the correct key according to the current epoch. This resolves the issue.

We present a unified construction in Algorithm 2.

Efficiency. Efficiency depends on the regime. We assume that Σ is instantiated using our construction from Section 4. In both regimes, storage and search communication is $O(1)$.

In the sparse regime, searches have $\widetilde{O}(\log N/p)+O(1)$ page efficiency: Each search query on a label ℓ with cnt matching values, at most $p-1$ distinct positions in T_γ of size $\mathsf{cap} = \widetilde{O}(p\log N/p)$ and at most $\mathsf{pgcnt} = \lfloor \mathsf{cnt}/p \rfloor$ full pages are accessed in Σ, where each Σ access induces $O(1)$ page accesses. Update page efficiency is $\widetilde{O}(\log N/p)$ as T_γ has size cap and Σ induces $\widetilde{O}(\log N/p)$ page accesses.

In this dense regime, tables T_γ are of size $\mathsf{cap} = p$. Search page is $O1$ and update page efficiency remains $\widetilde{O}(\log N/p)$.

Client storage. Note that in each epoch, there are at most M full pages that are completed and buffered in $\mathsf{CFP}_{\mathsf{new}}$. Therefore, client storage is $O(pM)$. This is simply a consequence of presenting the construction for both regimes in a unified manner to improve readability. With a simple modification, we obtain an upper bound $O(M) = M\cdot O(|\mathcal{V}|_b + |\mathcal{L}|_b + |\log N|)$ on client storage. We describe the modifcations below:

In the dense regime, there are at most M/p full pages amongst the values added in any epoch. Further, each bin T_γ contains at most $p-1$ values that match label ℓ such that $f(\ell) = \gamma$. Therefore, an incomplete page added in any epoch can lead to at most 1 full page when merged with the content in T_γ. Instead of adding this completed page to $\mathsf{CFP}_{\mathsf{new}}$, the full page is written to Σ immediately (or a dummy update is executed if no such page is completed), therefore there are at most $O(M/p)$ full pages are buffered on the client.

In the sparse regime, there are also at most N/p full pages amongst the fresh values per epoch. However, each bin T_γ can induce more than one completed page when merged with the incomplete pages in $\mathsf{CB}_{\mathsf{out}}[\gamma]$, and we cannot simply push all completed pages to Σ immediately without incurring an overhead in communication.

As a first step, let us argue that at most $\widetilde{O}(\log N/p)$ incomplete pages become full when a bin T_γ is merged with $\mathsf{CB}_{\mathsf{out}}[\gamma]$. If there were more than $\widetilde{O}(\log N/p)$ full pages in bin T_γ at any point during a merge, then this bin overflows over capacity $\mathsf{cap} = \widetilde{O}(\log N/p)$. This occurs only with negligible probability by the 1C upper bound (cf. lemma 9).

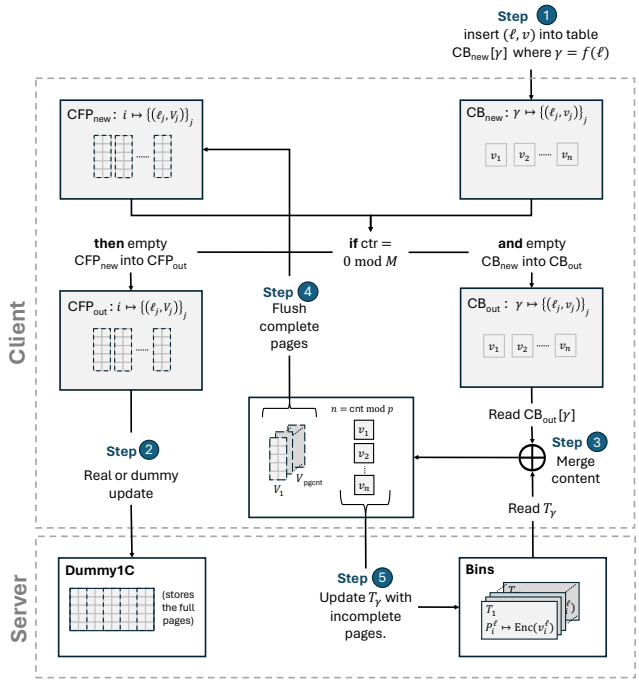


Figure 2 – An overview of our dynamic EMM scheme for the dense regime ($N \geq pM$). For each (ℓ, v) added to MM, the counter ctr, and the number of values associated with ℓ, $T_{\mathsf{len}}[\ell]$, are incremented. In this regime, there is one bin per label, each bin has capacity p, and each label ℓ maps to one table index γ via f.

We can reduce client storage as follows. Between each T_γ access, we postpone reading $T_{\gamma+1}$ for the next $\widetilde{O}(\log N/p)$ updates[3]. For each update, we perform 2 updates on Σ: one access to push a full page and another to (potentially) push one of the (up to) $\widetilde{O}(\log N/p)$ completed pages to Σ. After $\widetilde{O}(\log N/p)$-many updates after T_γ was read, all complete pages will be pushed to Σ before $T_{\gamma+1}$ is read. With this modification, the client storage is at most $O(M) + p \cdot \widetilde{O}(\log N/p) = O(M)$, as $N/p < M$ in this regime.

Correctness. It is straightforward to confirm correctness. Observe that all full pages are moved to Σ via the update mechanism and incomplete pages that match label ℓ are stored in bins $T_\gamma[f(\ell)]$. At most M pages are filled per epoch, therefore all full pages from $\mathsf{CFP}_{\mathsf{out}}$ are flushed to Σ each epoch.

Security. Setup leaks the upper bound N on the multi-map size and search leaks the query pattern qp and volume cnt. The scheme is forward and backward private (cf. Section 2.4.1). We give a formal proof in Appendix E.2.

[3]This is possible, since $\frac{M}{m} \geq \widetilde{O}(\log N/p)$ in this regime. Then, each epoch all tables T_γ will still be accessed and updated as before—instead of accessing all tables T_γ at the beginning of the epoch, the accesses of T_γ are spaced evenly amongst the M updates per epoch.

6 Evaluation

6.1 Experimental Setup

Choice of Primitives. We use HMAC-SHA256 with 32-byte keys as the PRF functions. We use HMAC-SHA256 without a key as the 1C hash functions. We use randomized AES-128-CBC as the encryption function in our implementation.

Implementation. We implement S1C and D1C in C++ with OpenSSL as the underlying cryptography library. The client and the server are implemented in the same program so there is no network layer in our experiments. We use standard C++ library functions (std::ifstream and std::ofstream) to read from and write to SSD. This means that, in the worst case, each encrypted page is stored in two physical pages by the operating system, leading to a factor of at most two in page efficiency. The implementation of S1C and D1C are publicly available at https://github.com/Merge-SSE/Merge-SSE.

Building Multi-maps. We pick $10K, 50K, 100K, 200K$ and $400K$ emails from the Enron email corpus to form raw databases. For each database, we use the Natural Language Toolkit[4] to extract the keywords from the emails. We build an inverted index mapping each keyword to the document identifiers of the documents containing the keyword and treat the inverted index as a multi-map where the labels are the keywords and the values are the matching document identifiers.

The resulting multi-maps are used directly in the experiments on S1C. For D1C, we further process the multi-maps so that they are either in the sparse regime ($N < pM$) or the dense regime ($N \geq pM$) (we determine p before running this; see Section 6.2). To obtain a multi-map in the sparse regime, we order the labels (ℓ_i) of the multi-map used for the experiment on S1C by the number of associated values ($|\mathsf{MM}[\ell_i]|$) in increasing order and add the labels to a new multi-map up until $N < pM$ no longer holds. Conversely, to obtain a multi-map in the dense regime, we order the labels of the multi-map used for the experiment on S1C by the number of associated values in decreasing order and continue adding the labels to a new multi-map until $N \geq pM$ no longer holds. The size of the multi-maps used in our experiments can be found in Table 2.

For the experiments used to determine the optimal page size (Section 6.2), we fixed δ to 3.5 to allow for fair comparison of the different page sizes. For all other experiments, we selected the smallest δ such that setup ran within a couple of minutes; we report the δ values used for each dataset in Table 2.

Concretely, we set the size of a label to 16 bytes ($|\mathcal{L}|_b = 16$ bytes) and the size of a value to 16 bytes ($|\mathcal{V}|_b = 16$ bytes) with appropriate padding.

Hash Table Instantiation. The hash tables used in S1C and D1C are both implemented as cuckoo tables. For S1C, the cuckoo tables are implemented with fingerprints. That is, for

[4]https://www.nltk.org/

$T_{\text{full}}[P_{i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j})$, the payload stored in the cuckoo table is $\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j}) \,||\, P_{i,j}$. As a consequence, when retrieving the entry with hash key $P_{i,j}$, the server can look up the entry in the clear (even though there are two possible locations for the payload). Thus, retrieving entries has optimal bandwidth.

In contrast, for D1C, the payload of $T_{\text{full}}[P_{i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j})$ is implemented as $\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j} \,||\, P_{i,j})$, thus, not exposing the key as described in Section D.2. The consequence is that the server must send two entries of a cuckoo table for each search query, resulting in a bandwidth overhead (and the client's computational overhead) of two.

The cuckoo tables must be instantiated with concrete capacities. For capacity cap in the pseudocodes (where $O(p\delta(\lambda)\log(N/p))$ is set to $3p\log(N/p)$ and $O(\delta(\lambda)\log(N))$ is set to $3\log(N)$), we set the size of the cuckoo tables to be $2(1+\delta)\mathsf{cap}$ (2 comes from the fact that a cuckoo table contains two hash tables and $(1+\delta)\mathsf{cap}$ is the number of entries in each hash table). The choice of δ mainly affects the setup time and update time (since we must encrypt more entries during setup and updates) and it does not have an effect on the query response time. We use different δ on different multi-maps and our choices of δ are reported in Table 2.

Experimental Environment. All benchmarks are executed on a single core of a machine with an AMD EPYC 7742 CPU @2.25 GHz with 64 cores and 512 GB of DDR4 memory. The machine has 2 x SATA III 3.84TB SSDs in RAID 1 connected via a Hardware RAID controller connected via PCI-E 3.0 x8.

6.2 Determining the Page Size

Before running the experiments, we need to determine the optimum page size p. We do this by measuring the search throughput of S1C and D1C on the full pages using the multi-map derived from 50K emails, with page sizes of 256, 512, 1024, 2048 and 4096 bytes. (here, δ is fixed to 2). These correspond to $p = 14, 30, 126, 254$ and 510 in our pseudocode. We query all keywords in the emails and report the throughputs in Fig. 3. We obtain the highest throughput with a page size equal to 512 bytes ($p = 30$). This is expected as most of the SSDs have 512-byte pages.[5] We use 512 bytes as the page size ($p = 30$) for all subsequent experiments.

Table 2 – General statistics of the multi-maps and δ values we used. N is the size of the multi-map. M is the number of labels. δ is the expansion factor of the cuckoo table. The two numbers in the rows for D1C are for the sparse regime ($N < pM$) and the dense regime ($N \geq pM$) respectively.

#emails	N	M	Scheme	δ
10K	276,206	11,505	S1C	0.7
	276,206/ 273,828	11,505 / 9,127	D1C	1.2 / 2.3
50K	1,650,241	18,510	S1C	2.1
	516,235/ 1,650,241	17,210 / 18,510	D1C	1.25 / 2.5
100K	3,222,025	22,153	S1C	2.5
	594,161 / 3,222,025	19,810 / 22,153	D1C	1.25 / 2.6
200K	7,579,886	28,287	S1C	3.9
	715,081 / 7,579,886	23,838 / 28,287	D1C	1.2 / 2.8
400K	13,950,793	32,112	S1C	4.3
	778,114 / 13,950,793	25,938 / 32,112	D1C	1.2 / 3

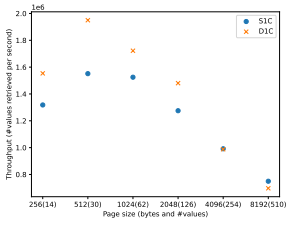


Figure 3 – Throughput of S1C and D1C using different page sizes. The optimal page size is 512 bytes (30 values per page) for both schemes on our machine.

6.3 Benchmarking S1C

Note that in the *static* setting and *dense* regime (*i.e.*, no updates, $pM \leq N$) there is a trivial solution: split lists into sub-lists of size p, pad the incomplete lists to size p and store all items in a non-page-efficient EMM (*e.g.*, [10]). In the dense *regime*, this solution has optimal performance and is implicit in our *dynamic* construction in the dense regime. We stress that *dynamic* and dense EMMs are non-trivial (cf. Section 1). Therefore, we do only evaluate S1C in the sparse regime.

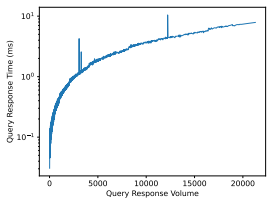
Setup Time. For $N = 276206, 1650241, 3222025, 7579886, 13950793$, S1C takes $2.71, 28.1, 12.3, 174.6, 72.6$ seconds to complete the setup, respectively. The setup time does not scale linearly, as building the cuckoo tables takes random amounts of time (due to reseeding when we fail to insert all elements into the table with a particular seed).

Storage. Due to space limitations, we only report client and server storage for the experiment on the largest dataset (400K emails). The client storage consists of 32 bytes of cryptographic keys. The server storage is 23.4 GB.

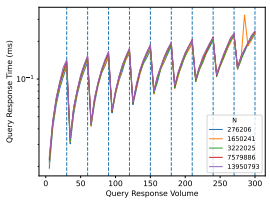
Query Time. We query all keywords in the 400K documents ($N = 13950793, M = 32112$) and depict the query performance of S1C in Fig. 4a. It only takes 7.87 milliseconds to retrieve $21,290$ values. We also conduct experiments on smaller multi-maps and depict the in Fig. 4b. The search time is not affected by the size of the multi-map and, as such, S1C should be considered practically efficient.

An interesting observation is that the query response time follows a zig-zag pattern (see Fig. 4b). Between query response volumes kp and $(k+1)p$ for some integer k and page

[5] The actual NAND flash memory page size of an SSD is often larger than 512 bytes, but the SSDs usually only expose their logical sector size as their "page size" to the operating system and the file system. The latter is typically 512 bytes. This explains why we observe a decreasing throughput as we set the page size of our experiments to be larger than 512 bytes, as a larger page size leads to more fragmented storage.



(a) Search performance of S1C on a multi-map with $N = 13950793, M = 32112$.



(b) Search performance of S1C on various multi-maps up to a query response volume of 300.

Figure 4 – Search performance of S1C.

size p, the query response time grows linearly. However, the query response time for a query with a query response volume slightly larger than $(k+1)p$ is generally smaller than the query response time of a query with a query response volume slightly smaller than $(k+1)p$. In other words, it is more efficient to retrieve a full page of values from T_{full} than to retrieve slightly less than p values from some $T_{\mathsf{val},\cdot}$. This serves as supporting evidence that S1C is page-efficient.

As an independent result, we run S1C in memory and observe the same zig-zag pattern on the query response time (see Fig. 7 in Appendix F). This is surprising as one would expect the use of pages to not have any effect on the query response time (as RAM does not have a page-like behavior). We believe that this result is due to the concrete efficiency gain when pages are used. For simplicity, assume that the length of each value is equal to the block size of the encryption scheme used (16 bytes in the case of AES-128-CBC). When encrypting p values as a single entry in T_{full}, the payload consists of $p+1$ ciphertext blocks (p blocks from the values and one block from the initialization vector) and one fingerprint block, so retrieving a full page in T_{full} takes one fingerprint check on the server, $p+2$ blocks of bandwidth, and p AES decryption operations on the client. On the other hand, if p values are encrypted individually, as in $T_{\mathsf{val},\cdot}$, we obtain p payloads, each containing two ciphertext blocks (one block from the value and one from the initialization vector) and one fingerprint block. Thus, retrieving p entries stored in $T_{\mathsf{val},\cdot}$ takes p fingerprint checks on the server, $3p$ blocks of bandwidth and p AES decryption operations on the client. Even without the use of fingerprints, retrieving query responses in pages is more bandwidth-efficient. We believe that our observation can help design more efficient in-memory SSE schemes.

6.4 Benchmarking D1C

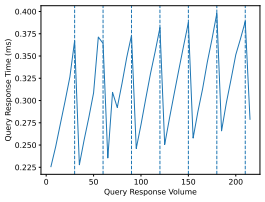
Setup Time. We use a modified setup procedure for D1C: instead of running setup with an empty multi-map, we run it with a non-empty multi-map and parameters p, N, and M. This modification allows us to benchmark the query responses without waiting for the label-value pairs to be inserted into the EMM one by one. The setup time with a non-empty multi-map is longer than that with an empty multi-map.

The setup time of D1C in the sparse regime ($N < pM$) for multi-maps of size between $N = 276206, M = 11505$ and $N = 778114, M = 25938$ ranges from 1.1 to 17.2 seconds. The setup time of D1C in the dense regime ($N \geq pM$) for multi-maps of size between $N = 273828, M = 9127$ and $N = 13950793, M = 32112$ ranges from 24.1 to 563.1 seconds. The setup time is longer in the dense regime as we used larger δ's in our experiments.

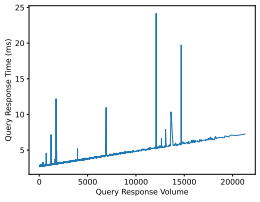
Storage. For 400K emails, in the sparse regime, the client storage consists of 64 bytes of cryptographic keys, 0.8 MB of constants and counters, and 0.8 MB of content in the stashes. The server storage is 0.42 GB. In the dense regime, the client storage consists of 64 bytes of cryptographic keys, 1 MB of constants and counters, and 1 MB of content in the stashes. The server storage is 4.3 GB.

Update Time. We ran 1000 update queries on the EMMs and reported the average update times. The update time ranged from 13.8 ms to 17.2 ms for the multi-maps in the sparse regime and ranges from 3.0 ms to 34.4 ms for the multi-maps in the dense regime. Updates required more time in the dense regime since larger δ values were used.

Query Time. We show the query response time of S1C on the multi-map derived from 400K documents ($N = 778114, M = 25938$ in the sparse regime; $N = 13950793, M = 32112$ in the dense regime) in Figures 5a and 5b, respectively. D1C appears to be more efficient in the sparse regime as the EMMs are smaller. So D1C benefits more from file system caching and caching in the SSD. Similar to S1C, D1C displays excellent search efficiency, taking only 0.40 milliseconds to retrieve 215 values in the sparse regime, and 24.2 milliseconds to retrieve 21285 values in the dense regime.



(a) Search performance on a multi-map with $N = 778114$, $M = 25938$. (sparse)



(b) Search performance on a multi-map with $N = 13950793$, $M = 32112$ (dense).

Figure 5 – Search performance of D1C.

Algorithm 2 D1C: pseudocode marked with ⌜ ⌝ and ▭ is executed in the sparse ($N < pM$) and dense regime ($N \geq pM$), respectively.

D1C.KeyGen(1^λ)

Sample $\mathsf{K_{Enc}}$ and $\mathsf{K_{PRF}}$ for Enc and F with λ, respectively
Sample $\mathsf{K_\Sigma} \leftarrow \Sigma.\mathsf{KeyGen}(1^\lambda)$
return $\mathsf{K} = (\mathsf{K_{Enc}}, \mathsf{K_{PRF}}, \mathsf{K_\Sigma})$

D1C.Setup(K, N, M)

Set ⌜ $\mathsf{cap} \leftarrow O(p\delta(\lambda)\log(N/p))$ ⌝ ▭ $\mathsf{cap} \leftarrow p$
Set ⌜ $m \leftarrow \left\lceil \frac{N}{p \cdot \delta(\lambda) \cdot \log(N/p)} \right\rceil$ ⌝ ▭ $m \leftarrow M$
⌜ Let $f : \mathcal{L} \to [m]$ such that $f(\ell) \mapsto \mathcal{H}_{1\mathsf{C}}(F_{\mathsf{K_{PRF}}}(\ell))$ ⌝
▭ Let $f : \mathcal{L} \to [m]$ be a random injective partial function ▷ implicitly built iteratively when a new label is added in Update
Initialize empty tables $T_{\mathsf{len}}, \mathsf{CB_{new}}, \mathsf{CB_{out}}, \mathsf{CFP_{new}}, \mathsf{CFP_{out}}$
for all $\gamma \in \{0, \ldots, m-1\}$ **do**
    Init empty table T_γ with cap entries, each of size $|\mathcal{V}|_b + |\mathcal{L}|_b$
    Set $T_\gamma[k_i] \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}(0\|0)$ for $k_i \leftarrow \{0,1\}^{2\lambda}, i \in [\mathsf{cap}]$
$\mathsf{ctr} \leftarrow 0$ ▷ Counts the total number of updates
$\mathsf{num} \leftarrow 0$ ▷ Counts the number of full pages added this epoch
$\pi \leftarrow S_M$ ▷ Sample random permutation
$\mathsf{EMM_{full}} \leftarrow \Sigma.\mathsf{Setup}(\mathsf{K_\Sigma}, \lceil N/p \rceil)$
$\mathsf{EMM} \leftarrow (\mathsf{EMM_{full}}, (T_0, \ldots, T_{m-1}))$
return EMM ▷ Client state is $(f, m, \mathsf{cap}, \mathsf{ctr}, \mathsf{num}, \pi, T_{\mathsf{len}}, \mathsf{CB_{new}}, \mathsf{CB_{out}}, \mathsf{CFP_{new}}, \mathsf{CFP_{out}})$

D1C.Search($\mathsf{K}, \ell; \mathsf{EMM}$)

Client:

Run $R_{\mathsf{full}} \leftarrow \Sigma.\mathsf{Search}(\mathsf{K_\Sigma}, \ell; \mathsf{EMM_{full}})$ ▷ retrieve full pages
Set $\gamma \leftarrow f(\ell)$ and $\mathsf{cnt} \leftarrow T_{\mathsf{len}}[\ell] \bmod p$
if $\mathsf{cnt} = \perp \vee \mathsf{cnt} = 0$ **then return** R_{full}
Set $\mathsf{epoch} \leftarrow \lceil \mathsf{ctr}/M \rceil$
if $b \leftarrow (\gamma > \mathsf{ctr} \bmod M)$ **then** ▷ T_γ was not updated this epoch
    Set $\mathsf{epoch} \leftarrow \mathsf{epoch} - 1$
Set $P_\ell \leftarrow F_{\mathsf{K_{PRF}}}(\mathsf{epoch}, \ell)$
send $(P_\ell, \mathsf{cnt}, \gamma)$

Server:

Initialize empty set R
for all $i \in \{1, \ldots, \mathsf{cnt}\}$ **do**
    Set $k_i \leftarrow \mathcal{H}_P(P_\ell, i)$ and add $T_\gamma[k_i]$ to R
send R

Client:

Decrypt R to retrieve incomplete list and combine with R_{full}
Retrieve values associated to ℓ from client state

D1C.Update($\mathsf{K}, (\ell, v), \mathsf{add}; \mathsf{EMM}$)

Client:

$\mathsf{ctr} \leftarrow \mathsf{ctr} + 1$
Add $(v\|\ell)$ to $\mathsf{CB_{new}}[\gamma]$ for $\gamma = f(\ell)$
Increment $T_{\mathsf{len}}[\ell] \leftarrow T_{\mathsf{len}}[\ell] + 1$
if $\mathsf{ctr} = 0 \bmod M$ **then** ▷ every fresh epoch
    Set $\mathsf{CB_{out}} \leftarrow \mathsf{CB_{new}}$ and empty $\mathsf{CB_{new}}$
    Set $\mathsf{CFP_{out}} \leftarrow \mathsf{CFP_{new}}$ and empty $\mathsf{CFP_{new}}$
    Set $\mathsf{num} \leftarrow 0$ and $\pi \leftarrow S_M$ ▷ sample random permutation
Set $\gamma \leftarrow \mathsf{ctr} - 1 \bmod M$
Set $(\ell_V, V) \leftarrow \mathsf{CFP_{out}}[\gamma + 1]$
if $(\ell_V, V) \neq \perp$ **then**
    Run $\Sigma.\mathsf{Update}(\mathsf{K}, (\ell_V, V), \mathsf{add}; \mathsf{EMM_{full}})$
else
    Run $\Sigma.\mathsf{DummyUpdate}(\mathsf{K}; \mathsf{EMM_{full}})$
if $\gamma \geq m$ **then return**
send γ

Server:

send T_γ

Client:

Set $V'_\gamma := (v'_i \| \ell'_i)_{i=1}^{\mathsf{cap}} \leftarrow \mathsf{Dec}_{\mathsf{K_{Enc}}}(T_\gamma)$
Remove $(0\|0)$ values from V'_γ
Add $\mathsf{CB_{out}}[\gamma]$ to V'_γ and empty $\mathsf{CB_{out}}[\gamma]$
Parse V'_γ as multi-map $\mathsf{MM}_\gamma = \{(\ell'_i, (v_1, \ldots, v_{n_{\ell'_i}}))\}_i$
Reinitialize T_γ as empty table with cap entries
Set $\mathsf{epoch} \leftarrow \lceil \mathsf{ctr}/M \rceil$
for all ℓ_i in MM_γ **do**
    Set $V_i \leftarrow \mathsf{MM}_\gamma[\ell_i]$ and $\mathsf{cnt}_i \leftarrow |V_i|$
    Set $\mathsf{pgcnt}_i \leftarrow \lfloor \mathsf{cnt}_i / p \rfloor$
    Partition V_i into $\mathsf{pgcnt}_i + 1$ lists $V_{i,j}$ such that $|V_{i,j}| = p$ for $j \leq \mathsf{pgcnt}_i$, and $0 \leq |V_{i,\mathsf{pgcnt}_i+1}| < p$
    for all $1 \leq j \leq \lfloor \mathsf{cnt}_i / p \rfloor$ **do**
        Set $\mathsf{num} \leftarrow \mathsf{num} + 1$
        Set $\mathsf{CFP_{new}}[\pi(\mathsf{num})] \leftarrow (\ell_i, V_{i,j})$
    Set $j \leftarrow 1$ and $P_i \leftarrow F_{\mathsf{K_{PRF}}}(\mathsf{epoch}, \ell_i)$
    for all $\mathsf{v}_j \in V_{i,\mathsf{pgcnt}_i+1}$ **do**
        Set $k_j \leftarrow \mathcal{H}_P(P_i, j)$ and $T_\gamma[k_j] \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}(\mathsf{v}_j \| \ell_i)$
        Set $j \leftarrow j + 1$
Fill T_γ up to capacity cap with fresh $\mathsf{Enc}_{\mathsf{K_{Enc}}}(0\|0)$ entries
Send T_γ

Server:

Update T_γ

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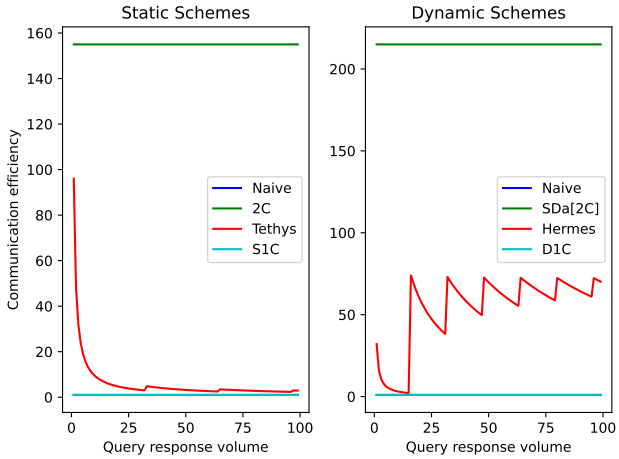
A Concrete Comparison

In this section, we provide concrete page efficiency and communication efficiency comparisons between S1C and prior works on static memory-efficient EMMs, and between D1C and prior works on dynamic memory-efficient EMMs, respectively.

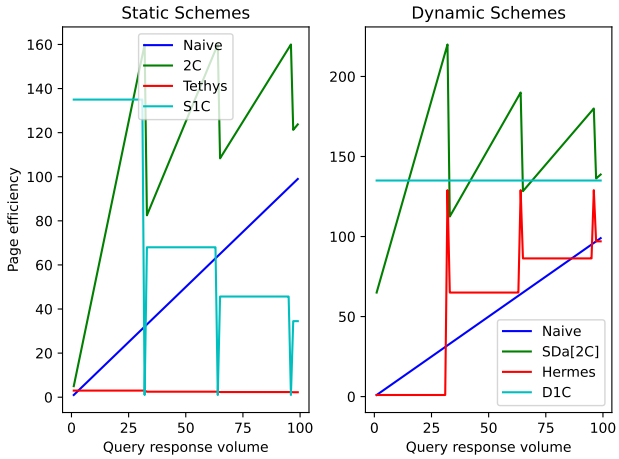
In these comparisons, we fix the size of the multimap to $N = 13,950,793$ and the page size to $p = 32$. And we compare the concrete overhead of the schemes with respect to the number of values retrieved in the query. The results will favour S1C and D1C even more if the page size is set to something larger (that is more common in modern SSDs).

We measure communication efficiency as the number of records retrieved by the given scheme for a query that is supposed to retrieve x records, divided by x. The comparison between selected schemes is shown in Figure 6a. The schemes that are omitted in the plots have worse communication efficiency than the ones that have been shown (e.g., 1C is worse than 2C). An optimal EMM should have communication efficiency 1 regardless of the number of records retrieved. The comparison shows that S1C and D1C are significantly better than the other schemes in terms of the communication efficiency.

We measure page efficiency as the number of pages accessed by the EMM for a query that is supposed to retrieve x records, divided by $\lceil x/p \rceil$, where p is the page size. The comparison between selected schemes is shown in Figure 6b. The schemes that are omitted in the plots have worse page efficiency than the ones that have been shown (e.g., 1C is worse than 2C). An optimal EMM should have page efficiency 1 regardless of the number of records retrieved. The comparison shows that while S1C and D1C are not optimal, they are practically acceptable.



(a) Communication efficiency.



(b) Page efficiency.

Figure 6 – Comparisons of communication efficiency and page efficiency between S1C, D1C, and selected schemes. We use $N = 13,950,793$ and $p = 32$ in the comparison.

B Theorems for Weighted One-Choice

We recall the weighted one-choice (1C) allocation process. Roughly, an arbitrary number of balls $\{b_i\}$ of weight $w_i \in [0,1]_{\mathbb{R}}$ with total weight at most N are thrown into m bins at random. Let $\delta(\lambda) = \log\log\lambda$. A detailed description is given in Algorithm 3, where $\mathcal{H}$ denotes a random oracle mapping into $\mathbb{Z}_m$ and bid is a unique identifier for each ball.

An $\widetilde{O}(\log N)$ upper bound for the most loaded bin in weighted one-choice is implicit in [5, 35]. For completeness, we give a proof below.

Algorithm 3 One-Choice Allocation (1C)

1C.Setup(N)

1: Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N))\rceil$
2: Initialize m empty bins $B_0,\ldots,B_{m-1}$
3: Return $B_0,\ldots,B_{m-1}$

1C.UpdateBall(bid, $w, B_0,\ldots,B_{m-1}$)

1: Receive bins $B_1,\ldots,B_m$, and ball bid with weight w
2: Set $\alpha \leftarrow \mathcal{H}(\text{bid})$
3: **if** bid $\notin B_\alpha$ **then**
4: Insert bid into B_α
5: Update weight of bid to w in B_α

Lemma 9 (1C). *Let $N = \text{poly}(\lambda)$, $\delta(\lambda) = \log\log\lambda$, and $m = \lceil N/(\delta(\lambda)\log(N))\rceil$. Assume $m = \Omega(\lambda)$. Then the most loaded bin of M1C during the execution of a sequence of* UpdateBall *operations has load at most $O(\delta(\lambda)\log N)$ except with negligible probability, if the total weight of the inserted balls does not exceed N.*

We first recall some useful lemmas. The first is one of the formulations of the Chernoff bound. The second lemma is a first-moment argument borrowed from [35]. The third lemma gives upper bounds on the most-loaded bin in weighted one-choice process in expectation.

Lemma 10 (Chernoff's Bound)**.** *Suppose that $X_1,\ldots,X_n$ are independent random variables taking values in $\{0,1\}$. Let X denote their sum and let $\mu = \mathbb{E}(X)$ denote the expectancy of X. Then for any $\delta > 0$, it holds that*

$$\Pr[X \geq (1+\delta)\mu] \leq \exp\left(-\frac{\delta^2\mu}{2+\delta}\right)$$

Lemma 11 ([35])**.** *Let $N \in \mathbb{N}$ with $N = \text{poly}(\lambda)$. For any integer-valued random variable $X \in [0,N]$ and any $R_N > 0$:*

$$\Pr[X > R_N] = \text{negl}(\lambda) \quad \textit{iff} \quad \mathbb{E}(\max(X - R_N, 0)) = \text{negl}(\lambda).$$

Lemma 12 ([5], Corollary 3.5)**.** *Let $N \in \mathbb{N}$ and $c \in [0,1]_{\mathbb{R}}$. Let $\tilde{w} = (c)_{i\in[\tilde{n}]}$ and $w = (w_i)_{i\in[n]}$ be non-negative vectors. Let $\sum_{i=1}^{n} w_i \leq c\cdot\tilde{n}$ and $w_i \leq c$ for all $i \in [n]$.*

Denote by X_{mlb} (resp. $\tilde{X}_{\text{mlb}}$) the random variable indicating the load of the most loaded bin after throwing n balls with weights w (resp. N balls with weights $\tilde{x}$) uniformly and independently at random into m bins. For any positive $R \in \mathbb{R}$, it holds that

$$\mathbb{E}[\max(X_{\text{mlb}} - R, 0)] \leq \mathbb{E}[\max(\tilde{X}_{\text{mlb}} - R, 0)].$$

Note that we simplified the formulation of [5] and adapted it to our needs. Roughly, [5] requires that $\tilde{w}$ *majorizes* w, which holds if both are chosen as in Lemma 12. Then, [5] shows that $\mathbb{E}[\tilde{X}_{\text{mlb}}] \geq \mathbb{E}[X_{\text{mlb}}]$ using a convexity argument. As $f(X) = \max(X - R, 0)$ is increasing, it is straightforward to adapt their proof to our formulation. For the upper bound on the most loaded bin in weighted 1C, our formulation is sufficient. We are now ready to prove the upper bound.

Proof of Lemma 9. Let X_{mlb} be the random variable denoting the load of the most loaded bin at some point during the sequence. Let $R = 3\lceil N\rceil/m$. We show that $\Pr[X_{\text{mlb}} > R+1] = \text{negl}(\lambda)$. Note that this is implied by $\Pr[\lceil X_{\text{mlb}}\rceil > R+1] = \text{negl}(\lambda)$.

As $\lceil X_{\text{mlb}}\rceil < \lceil N\rceil$ is integer-valued, we can apply Lemma 11. Now, we need to show that $\mathbb{E}(\max(\lceil X_{\text{mlb}}\rceil - (R+1), 0)) = \text{negl}(\lambda)$. As $\lceil X_{\text{mlb}}\rceil < X_{\text{mlb}} + 1$, it suffices to show that the expression $\mathbb{E}(\max(X_{\text{mlb}} - R, 0))$ is negligible in λ.

By Lemma 12, this quantity is upper bounded by $\mathbb{E}[\max(\tilde{X}_{\text{mlb}} - R, 0)]$, where $\tilde{X}_{\text{mlb}}$ is the load of the most loaded bin for $\lceil N\rceil$ balls of weight 1. Note that $\tilde{X}_{\text{mlb}} \leq \lceil N\rceil$ is a non-negative integer variable. After a final application of Lemma 11, it remains to show that $\Pr[\tilde{X}_{\text{mlb}} \geq R] = \text{negl}(\lambda)$. For the last quantity, it is sufficient to analyze classical 1C without weights.

Let X_i be the random variable that denotes the number of balls in bin B_i. Every ball has a chance of $1/m$ to be assigned to B_i and there are $\lceil N\rceil$ balls. Thus, we have $\mathbb{E}[X_i] = \lceil N\rceil/m$. Note that X_i can be expressed as sum of independent random variables taking values in $\{0,1\}$. Applying Lemma 10 with constant 2, we obtain:

$$\Pr[X_i \geq 3\,\mathbb{E}[X_i]] \leq \exp\left(-\frac{4\cdot\mathbb{E}[X_i]}{4}\right)$$
$$\Longrightarrow \Pr[X_i \geq R] \leq \exp(-\lceil N\rceil/m) = \text{negl}(\lambda),$$

by assumption. Finally, a union bound over all bins yields that $\Pr[\tilde{X}_{\text{mlb}} \geq R] = \text{negl}(\lambda)$. The statement follows because

$$\exp(-\delta(\lambda)\log N) = \text{negl}(\lambda)$$

if $N \geq \lambda$ and $\delta(\lambda) = \omega(1)$. □

C Details for our Memory-Efficient EMMs

In this section, we provide detailed overviews of our constructions in Sections 3 and 5.

C.1 Detailed Description of our Static Scheme

Let us give a detailed overview of the scheme in Section 3. A description in pseudocode is given in Algorithm 1.

S1C makes use of two hash functions: a token hash function $\mathcal{H}_P : \{0,1\}^* \to [2^{2\lambda}]$ that maps to the key space of T_{full} and a one-choice hash function $\mathcal{H}_{1\mathsf{C}} : \{0,1\}^* \to [0, m-1]$ that maps to the space of bin identifiers. The scheme also makes use of a PRF, denoted PRF, that maps to the space $\{0,1\}^{2\lambda} \times \{0,1\}^{\lceil\log N\rceil} \times \{0,1\}^{2\lambda}$.

To encrypt a multimap MM, the client starts by executing the KeyGen algorithm, which takes as input security parameter λ and which samples a PRF key, $\mathsf{K}_{\mathsf{PRF}}$, and an encryption key, $\mathsf{K}_{\mathsf{Enc}}$.

Next, the client executes Setup, which takes as input the security parameter λ, secret key K, upper bounds N and M, and the multimap MM. The client initializes two tables: T_{len} with N entries each of size $\lceil \log N \rceil$, and T_{full} with $\lceil N/p \rceil$ entries each of size $p \cdot |\mathcal{V}|_b$. It computes $m \leftarrow \lceil N/(p \cdot \delta(\lambda) \cdot \log(N/p)) \rceil$ and $\mathsf{cap} \leftarrow O(p \cdot \delta(\lambda) \log(N/p))$. Then, it initializes m tables, $T_{\mathsf{val},\gamma}$ for $\gamma \in [0, m-1]$, each with cap random entries.

For each label $\ell_i \in \mathsf{MM}$, the client retrieves the associated values $V_i = \mathsf{MM}[\ell_i]$ and computes the number of associated values, $\mathsf{cnt}_i = |V_i|$, and the number of pages needed to stores those values, pgcnt_i. The search tokens K_i and P_i' (for T_{len} and T_{full}, respectively), and the mask μ_i are then derived by evaluating the PRF on the label, i.e., $(K_i, \mu_i, P_i') \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\ell_i)$. For each label $\ell_i \in \mathsf{MM}$, the client retrieves the corresponding value set $V_i = \mathsf{MM}[\ell_i]$ and computes its cardinality, $\mathsf{cnt}_i = |V_i|$, and the number of pages needed to stores those values, $\mathsf{pgcnt}_i = \lceil \mathsf{cnt}_i/p \rceil$. The search token K_i (for T_{len}) , the mask μ_i, and the search token P_i' (for T_{full}) are then derived by evaluating the PRF on the label, i.e., $(K_i, \mu_i, P_i') \leftarrow \mathsf{PRF}_{\mathsf{K}_{\mathsf{PRF}}}(\ell_i)$.

The number of values is blinded and stored as $T_{\mathsf{len}}[K_i] \leftarrow \mathsf{cnt}_i \oplus \mu_i$. This will later be used at query time to help the server determine how many values must be retrieved. The values associated with label ℓ_i are partitioned into pgcnt_i many lists, which we denote as $V_{i,j}$; each list is of length p (the page size). Then, for all $j = 1, \ldots, \lfloor \mathsf{cnt}_i/p \rfloor$, we compute the hash output $P_{i,j} \leftarrow \mathcal{H}_P(P_i', j)$ and store the encrypted full pages as $T_{\mathsf{full}}[P_{i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(V_{i,j})$.

It remains to compute the tables $T_{\mathsf{val},0}, \ldots, T_{\mathsf{val},\mathsf{m}-1}$. For each label ℓ_i, the client computes the hash output $P_{i,\mathsf{pgcnt}_i} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i)$ and sets $\gamma \leftarrow \mathcal{H}_{1\mathsf{C}}(P_{i,\mathsf{pgcnt}_i})$. The value γ specifies which bin to insert the corresponding values into. The client initialzes $j \leftarrow 1$. Then, for each value v in the final partition V_{i,pgcnt_i}, the client: (1) computes the location $P_{i,\mathsf{pgcnt}_i,j} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i, j)$, (2) encrypts the value and stores it in the table as $T_{\mathsf{val},\gamma}[P_{i,\mathsf{pgcnt}_i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(\mathsf{v})$, and (3) increments j.

Setup concludes with the client filling the remaining empty entries in T_{full} and T_{len} with random values and outputing $\mathsf{EMM} = (T_{\mathsf{len}}, T_{\mathsf{full}}, (T_{\mathsf{val},0}, \ldots, T_{\mathsf{val},\mathsf{m}-1}))$ to the server.

To make a query for a label ℓ_i, the client and server jointly execute Search. The search protocol takes the private key K and the label ℓ_i as input from the client, and EMM as input from the server. The client starts by computing the search token as $(K_i, \mu_i, P_i') \leftarrow \mathsf{PRF}_{\mathsf{K}_{\mathsf{PRF}}}(\ell_i)$ and returning it to the server.

On input of the search token, the server retrieves the (encrypted) values as follows. First it initializes an empty set R that will store the response. The number of values is retrieved as $\mathsf{cnt}_i \leftarrow T_{\mathsf{len}}[K_i] \oplus \mu_i$ and cnt_i is then used to compute pgcnt_i. For each $j = 1, \ldots, \lfloor \mathsf{cnt}_i/p \rfloor$, the hash $P_{i,j} \leftarrow \mathcal{H}_P(P_i', j)$ is computed and the full (encrypted) page $T_{\mathsf{full}}[P_{i,j}]$ is added to R. The server then uses P_i' and pgcnt_i to derive P_{i,pgcnt_i} and γ. Then, for each $j \in [\mathsf{cnt}_i \bmod p]$, it computes the value's location $P_{i,\mathsf{pgcnt}_i,j} \leftarrow \mathcal{H}_P(P_i', \mathsf{pgcnt}_i, j)$ and adds $T_{\mathsf{val},\gamma}[P_{i,\mathsf{pgcnt}_i,j}]$ to R. The server returns R to the client.

Finally, the client decrypts the values in R using $\mathsf{K}_{\mathsf{Enc}}$.

C.2 Detailed Description of our Dynamic EMM

Our dynamic, page-efficient EMM scheme operates in two regimes, however we describe them as one unified scheme.

Similar to our static scheme, D1C makes use of two hash functions: a token hash function $\mathcal{H}_P : \{0,1\}^* \to [2^{2\lambda}]$ that maps to the key space of the bins and a one-choice hash function $\mathcal{H}_{1\mathsf{C}} : \{0,1\}^* \to [0, m-1]$ that maps to the space of bin identifiers. The scheme also uses a PRF,PRF, that maps to the space $\{0,1\}^{2\lambda} \times \{0,1\}^{\lceil \log N \rceil} \times \{0,1\}^{2\lambda}$. It also makes black-box use of a EMM scheme with dummy updates, which we denote by Σ.

Key generation. To encrypt a multimap MM, the client must first generate the secret key using KeyGen. This algorithm takes as input the security parameter λ and samples a PRF key, $\mathsf{K}_{\mathsf{PRF}}$ and an encryption key, $\mathsf{K}_{\mathsf{Enc}}$. It also executes $\Sigma.\mathsf{KeyGen}(1^\lambda)$ to generate the key K_Σ for the underlying EMM. It returns $(\mathsf{K}_{\mathsf{Enc}}, \mathsf{K}_{\mathsf{PRF}}, \mathsf{K}_\Sigma)$ to the client.

Setup. To setup the EMM, the client executes Setup which takes as input the secret key K and upper bounds on the EMM size, N, and number of labels, M. It is at this point that the scheme description for the two regimes diverges:

Regime $N < pM$: In this regime we define the function f as a one-choice allocation ($1\mathsf{C}$) function; f is used to select which bin to store the values of a given label. Each bin is of size $\mathsf{cap} = p \cdot \widetilde{O}(\log N/p)$ and there are $m = O(N/\mathsf{cap})$ bins in total.

Regime $N \geq pM$: In contrast, this regime selects the bin index using an injective choice function f. Each of size $\mathsf{cap} = p$ and there are $m = M$ bins in total.

From here the two different regimes proceed similarly. The client initializes five tables which are all stored client-side as part of the state: T_{len} which stores the number of values associated with each label, two client buffers, $\mathsf{CB}_{\mathsf{new}}$ and $\mathsf{CB}_{\mathsf{out}}$, and two buffers to store full pages, $\mathsf{CFP}_{\mathsf{new}}$ and $\mathsf{CFP}_{\mathsf{out}}$. Next, the client initializes m bins, $\{T_i\}_{m \in [0,m-1]}$, and fills each of them with cap many encryptions of zeros, i.e., $\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0\|0)$. The client then initializes a counter that maintains the total number of life-time updates, $\mathsf{ctr} = 0$, a counter that maintains the number of full pages added in the current epoch, $\mathsf{num} = 0$, and samples a random permutation π on M. Finally, if runs $\Sigma.\mathsf{Setup}$ to obtain an (empty) EMM, $\mathsf{EMM}_{\mathsf{full}}$,

which will store the full pages on the server side and sends $\mathsf{EMM} = (\mathsf{EMM}_{\mathsf{full}}, (T_0, \ldots, T_{m-1}))$ to the server.

Update. To add a label-value pair (ℓ, v), to the EMM the client and server execute the Update protocol as follows. On the client-side, the counter ctr is incremented and the pair is added to the client buffer as $\mathsf{CB}_{\mathsf{new}}[\gamma] \leftarrow (\ell, v)$ for $\gamma = f(\ell)$ (recall, that f is defined as the one choice function for regime $N < pM$ and as an injective function otherwise). The number of values $T_{\mathsf{len}}[\ell]$ is also incremented.

If $\mathsf{ctr} \equiv 0 \bmod M$, then a new epoch starts; the client empties buffers $\mathsf{CB}_{\mathsf{new}}$ and $\mathsf{CFP}_{\mathsf{new}}$ into $\mathsf{CB}_{\mathsf{out}}$ and $\mathsf{CFP}_{\mathsf{out}}$, respectively, re-initializes $\mathsf{num} = 0$, and samples a fresh permutation π.

It sets $\gamma \equiv \mathsf{ctr} - 1 \bmod M$ and retrieves $(\ell, V) \leftarrow \mathsf{CFP}_{\mathsf{out}}[\gamma + 1]$. If this pair is non-empty, (i.e., there is a full page to be flushed to Σ) then we run Σ.Update to add (ℓ, V) to $\mathsf{EMM}_{\mathsf{full}}$. Otherwise, the pair is empty and we execute a dummy query via Σ.DummyUpdate. If $f \geq m$ then return. Finally, the client sends the index γ to the server.

The server responds by sending the bin T_γ back to the client who first decrypts the bin as $V'_\gamma := (v'_i \| \ell'_i)_{i=1}^{\mathsf{cap}} \leftarrow \mathsf{Dec}_{\mathsf{K}_{\mathsf{Enc}}}(T_\gamma)$ and removes any padding of the form $0\|0$. The contents of $\mathsf{CB}_{\mathsf{out}}$ are then emptied into V'_γ and V'_γ is parsed as a multimap $\mathsf{MM}_\gamma = \{(\ell'_i, (v_1, \ldots, v_{n_{\ell'_i}}))\}_i$. Bin T_γ is re-initialzed as an empty table with cap entries and the epoch number is set to $\mathsf{epoch} \leftarrow \lceil \mathsf{ctr}/M \rceil$.

For each label $\ell_i \in \mathsf{MM}_\gamma$ the client does the following. It retrieves the corresponding values V_i, sets the total number of values as $\mathsf{cnt}_i = |V_i|$, and sets the total page count as $\mathsf{pgcnt}_i = \lfloor \mathsf{cnt}_i / p \rfloor$. The set of values V_i is partitioned into full pages $V_{i,j}$ each of size p, for $j \leq \mathsf{pgcnt}_i$ and (potentially) a final, incomplete page $V_{i,\mathsf{pgcnt}_{i+1}}$. For each full page $V_{i,j}$, we increment num and insert the full page and its corresponding label as $\mathsf{CFP}_{\mathsf{new}}[\pi(\mathsf{num})] \leftarrow (\ell_i, V_{i,j})$. For each value v_j in the incomplete page, if it exists, we derive the value $k_i \leftarrow \mathcal{H}_P(P_i, j)$ and insert the encrypted value-label pair into the bin as $T_\gamma[k_j] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(v_j \| \ell_i)$.

The client concludes by padding up T_γ with fresh $\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0\|0)$ encryptions and sending it back to the server who then updates EMM accordingly.

Search. To query the EMM for a label ℓ, the client and server execute the following protocol. The client starts by retrieving all of the full pages from the EMM with dummy updates, Σ, i.e., $R_{\mathsf{full}} \leftarrow \Sigma.\mathsf{Search}(\mathsf{K}_\Sigma, \ell; \mathsf{EMM}_{\mathsf{full}})$. It then computes the bin index $\gamma \leftarrow f(\ell)$ and computes the number of values stored in bin T_γ as $\mathsf{cnt} \leftarrow T_{\mathsf{len}}[\ell] \bmod p$.

If $\mathsf{cnt} = \perp$ or $\mathsf{cnt} = 0$, then there are no additional values to retrieve and the client simply returns R_{full}. Otherwise, it computes $\mathsf{epoch} \leftarrow \lceil \mathsf{ctr}/M \rceil$. If $(\gamma > \mathsf{ctr} \bmod M)$, then T_γ was not updated this epoch and we decrement epoch. We then derive the search token, $P_\ell \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\mathsf{epoch}, \ell)$, and send $(P_\ell, \mathsf{cnt}, \gamma)$ to the server.

The server initializes an empty set R and for all $i \in [\mathsf{cnt}]$, computes $k_i \leftarrow \mathcal{H}_P(P_\ell, j)$, retrieves the encrypted pair from $T_\gamma[k_i]$, and adds it to R. When all encrypted pairs have been retrieved, the server sends R to the client. To conclude, the client decrypts R, combines it with R_{full}, and retrieves andy remaining values associated with ℓ from the client state.

D Dummy1C

In this section, we formalize our EMM scheme with dummy updates, Dummy1C, which supports $\mathsf{poly}(\lambda)$ dummy updates with $\widetilde{O}(\log N)$ communication for updates and *constant* search communication. This gives a different tradeoff to $\mathsf{Dummy}(\Sigma)$ [36] which has $\widetilde{O}(\log\log N)$ communication for both updates and searches.

We first give a formal definition of EMM with dummy updates. Then, we recap the framework in [36] and describe our construction based on our keyless cuckoo hash tables (Appendix D.2).

D.1 Security of EMMs with Dummy Updates

The notion of searchable encryption supporting dummy updates was introduced in [36]. We adapt their definition to EMMs below.

An EMM schemes supports dummy updates if its interface is equipped with an additional protocol, DummyUpdate, which takes as input only the client's master key K. The Setup algorithm additionally receives a parameter D, which specifies an upper bound on the total number of dummy updates. We omit unecessary parameters for simplicity.

Intuitively, an EMM scheme with dummy updates is secure if it is secure in the same sense as a normal EMM scheme with the added requirement that dummy updates should be indistinguishable from real updates from the server's perspective.

Definition 13 (Adaptive Semantic Security with Dummy Updates)**.** *Let Σ be an EMM scheme supporting dummy updates. Let $q \in \mathbb{N}$ be such that $q = \mathsf{poly}(\lambda)$ and $\mathcal{L} = (\mathcal{L}_{\mathsf{Stp}}, \mathcal{L}_{\mathsf{Srch}}, \mathcal{L}_{\mathsf{Updt}})$ be a leakage function. We say that Σ is $\mathcal{L}$-adaptively secure with support for dummy updates if for all honest-but-curious PPT adversaries $\mathcal{A}$, there exists a (stateful) PPT simulator Sim such that*

$$|\Pr[\mathrm{EMMREAL}^{\mathsf{d,adp}}_{\Sigma,\mathcal{A}}(\lambda) = 1] - \Pr[\mathrm{EMMIDEAL}^{\mathsf{d,adp}}_{\Sigma,\mathsf{Sim},\mathcal{L}}(\lambda) = 1]| = \mathsf{negl}(\lambda),$$

where the games $\mathrm{EMMREAL}^{\mathsf{d,adp}}_{\Sigma,\mathcal{A}}$ and $\mathrm{EMMIDEAL}^{\mathsf{d,adp}}_{\Sigma,\mathcal{A},\mathcal{L}}$ are defined in Algorithm 4.

Algorithm 4 Semantic security of dummy updates (Definition 13).

$\text{EMMREAL}^{\mathsf{d,adp}}_{\Sigma,\mathcal{A}}$

$(N,M,D,\mathsf{MM},\mathsf{st}_{\mathcal{A}}) \leftarrow \mathcal{A}(1^{\lambda})$
$\mathsf{K} \leftarrow \Sigma.\mathsf{KeyGen}(1^{\lambda})$
$\mathsf{EMM} \leftarrow \Sigma.\mathsf{Setup}(\mathsf{K},N,M,D,\mathsf{MM})$
Send EMM to $\mathcal{A}$
for $1 \le i \le q$ **do**
  $(\mathsf{op}_i,\mathsf{in}_i,\mathsf{st}_{\mathcal{A}}) \leftarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
  if $\mathsf{op}_i = \mathsf{srch}$ **then**
    Parse $\mathsf{in}_i = \ell_i$
    $\Sigma.\mathsf{Search}_C(\mathsf{K},\ell_i) \leftrightarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
  else if $\mathsf{op}_i \in \{\mathsf{add},\mathsf{del}\}$ **then**
    Parse $\mathsf{in}_i = (\ell_i,v_i)$
    $\Sigma.\mathsf{Update}_C(\mathsf{K},\ell_i,v_i,\mathsf{op}_i) \leftrightarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
  else
    $\Sigma.\mathsf{DummyUpdate}_C(\mathsf{K}) \leftrightarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
Output bit $b \leftarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$

$\text{EMMIDEAL}^{\mathsf{d,adp}}_{\Sigma,\mathsf{Sim},\mathcal{L},\mathcal{A}}$

$(N,M,D,\mathsf{MM},\mathsf{st}_{\mathcal{A}}) \leftarrow \mathcal{A}(1^{\lambda})$
$\mathsf{EMM} \leftarrow \mathsf{Sim}(\mathcal{L}_{\mathsf{Stp}}(\mathsf{MM},N,M,D))$
Send EMM to $\mathcal{A}$
for $1 \le i \le q$ **do**
  $(\mathsf{op}_i,\mathsf{in}_i,\mathsf{st}_{\mathcal{A}}) \leftarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
  if $\mathsf{op}_i = \mathsf{srch}$ **then**
    $\mathsf{Sim}(\mathcal{L}_{\mathsf{Srch}}(\mathsf{in}_i)) \leftrightarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
  else
    $\mathsf{Sim}(\mathcal{L}_{\mathsf{Updt}}(\mathsf{op}_i,\mathsf{in}_i)) \leftrightarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$
Output bit $b \leftarrow \mathcal{A}(\mathsf{st}_{\mathcal{A}})$

D.2 Keyless Cuckoo Hash Table

To address this search communication blowup, we propose a new EMM scheme, Dummy1C (described in Section D.3), which leverages a new data-structure: keyless cuckoo hash tables. We first recap cuckoo hashing which serves as the basis for our hash table.

Cuckoo hashing [40]. In cuckoo hashing, n items are inserted into two cuckoo tables T_1, T_2 each of capacity $(1+\varepsilon)n$, for some arbitrary constant $\varepsilon > 0$.

Each table cell can contain at most one item. For each item to be inserted, two positions $\alpha,\beta \leftarrow \{0,\ldots,(1+\varepsilon)n-1\}$ are chosen independently and uniformly at random (e.g., by hashing a value of the item). Each value, bid, is then inserted into either $T_1[\alpha]$ or $T_2[\beta]$ according to the following process: if one of the cells $T_1[\alpha]$ or $T_2[\beta]$ is free, bid is placed in the free cell, else bid replaces the exisiting item bid$'$ in $T_1[\alpha]$ and bid$'$ is moved to its other possible cell, potentially creating a chain reaction. The probability that all n items could be inserted is bounded by $O(1/n)$. In case of failure, new hash functions are chosen and the allocation process is restarted; this event is called a *rehash*. Pagh and Rodler show that cuckoo hashing allocates n items in expected time $O(n)$, including the cost of rehashes.

Lemma 14 (Cuckoo-Hashing [40]). *For $n \in \mathbb{N}$ and $\varepsilon > 0$, the runtime of allocating n items in cuckoo tables T_1, T_2 of capacity $(1+\varepsilon)n$ via the cuckoo process is $O(n)$ in expectation, and the probability that allocation fails is at most c for some constant $c < 1$, where the probability is over the random positions choices of each ball.*

For a more detailed description of cuckoo hashing, see [40].

Keyless hash table construction. For our EMM construction, we need a hash table with constant lookups (in the worst case) and a setup that leaks no information about the keys of the input items. Cuckoo-hashing fulfills these requirements, so long as the keys are not stored together with the items within the hash table. However, if the keys are not stored with the items, then the key of the searched item cannot be compared with the key of the two possible items during lookup. We thus simply return both items and the client can later decide which was the right value.

If we were to use cuckoo hashing with a stash [33], then setting up the cuckoo table would have constant overhead at the cost of a logarithmic stash. Unfortunately, since we employ multiple hash tables within our EMM constructions, the resulting client storage due to stashes would become prohibitive. To overcome this, we rehash keys by choosing new seeds for the hash function until allocation is possible. We note that this seemingly minor change introduces some technical challenges in the security proof our constructions later on.[6]

We now formally describe our hash table variant, CT. Let $\varepsilon > 0$ and let $\mathcal{H}_{\mathsf{CT}} : \{0,1\}^* \to [1,m]$ be a hash function.

- $\mathsf{CT.HTSetup}(S)$: On input of n key-value pairs $S = (k_i,x_i)_{i=1}^n$, do the following. Sample $\mathsf{seed} \leftarrow \{0,1\}^{2\lambda}$ for $\mathcal{H}_{\mathsf{CT}}$. Generate tables T_1, T_2 of size $n(1+\varepsilon)$ each, initialized with zero entries. Compute $\alpha_i,\beta_i \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed},k_i)$. Insert x_i at either $T_1[\alpha_i]$ or $T_2[\beta_i]$ according to the cuckoo process (see above). If allocation is not possible, choose new seed $\mathsf{seed} \leftarrow \{0,1\}^{2\lambda}$ and retries. If allocation was not successful after λ tries, output $\perp$. Otherwise, set $T = (T_1,T_2)$ and return (seed,T). Optionally, also output all seeds S_{seeds} that

[6] Some applications that use cuckoo hashing cannot afford to rehash, e.g., Tethys is insecure when rehashing is performed [6]. Through careful analysis we are able to show that–in our case–the rehashing procedure does not leak additional information.

were tried in the above procedure.[7]

- $\mathsf{CT.HTLookup}(k_i, T, \mathsf{seed})$: Given key k_i, tables $T = (T_1, T_2)$ and seed, return $T_1[\alpha_i], T_2[\beta_i]$ for $\alpha_i, \beta_i \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}, k_i)$.

Lemma 15. *We model $\mathcal{H}_{\mathsf{CT}}$ as a random oracle. The probability that setup fails,* i.e., $\perp \leftarrow \mathsf{CT.HTSetup}(S)$*, is negligible. The expected runtime of* $\mathsf{CT.HTSetup}(S)$ *is $O(|S|)$ and the runtime of* $\mathsf{CT.HTLookup}(k_i, T, \mathsf{seed})$ *is $O(1)$ for all inputs* (k_i, T, seed).

Proof. This is an immediate consequence of Lemma 14. The probability that allocation fails for some seed is at most $c < 1$ and, thus, the probability that allocation fails for λ seeds is at most $O(c^{\lambda}) = \mathsf{negl}(\lambda)$, as the hash functions values are independently random due to different seeds. □

In the following, we omit CT in CT.HTSetup and CT.HTLookup if the use of CT is clear by context.

Remark. The static volume-hiding EMM scheme of [41] also uses cuckoo hashing with a stash and similarly returns both items at the two hash locations for a given searched item. However, they do so with different motivations. In their case, updates are of no concern; since the items in the tables are encrypted using the client's secret key, the client simply requests both items from the server. Unlike our keyless hash table, they do not hash using a seed during setup.

D.3 Detailed Description

We refer to Section 4 for a brief overview of $\mathsf{Dummy}(\Sigma)$ [36], which our construction is based on, and a brief high-level description of our construction Dummy1C. Below, we describe Dummy1C in detail.

Let $\delta(\lambda) \leftarrow \log\log(\lambda)$, $m \leftarrow O(N/(\delta(\lambda)\log(N)))$ be the number of tables and $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$ be the table capacity. Let $\mathcal{H}_{1\mathsf{C}}$ be a hash function mapping to $\{0,\dots,m-1\}$. This hash function will be used for the underlying 1C mechanism. Similarly, let $\mathcal{H}_{\mathsf{CT}}$ be a hash function mapping to $\{0,\dots,(1+\varepsilon)\mathsf{cap}\}^2$ for some $\varepsilon > 0$. We use $\mathcal{H}_{\mathsf{CT}}$ for keyless cuckoo hash tabes CT of capacity U (cf. Appendix D.2). Roughly, the server stores the multi-map values in m CT hash tables $T_0,\dots,T_{m-1}$. The table where some (encrypted) value is stored is derived via 1C, which ensures that the tables T_γ need to accommodate at most $\mathsf{cap} = \widetilde{O}(\log n)$ items and allows for dummy updates. We give a description of Dummy1C below; a formal description of Dummy1C is given in Algorithm 5. Note that we assume that Setup does not obtain a multi-map MM as input for simplicity: As the scheme is forward-private, the database can be setup via Update operations online.

During setup, the client first sets up the hash tables $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$ for all $\gamma \in \{0,\dots,m-1\}$, initially with random input $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$. She then encrypts $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma)$ and sends the (empty) encrypted multi-map $\mathsf{EMM} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$ to the server. Later, the key-value pairs in S_γ will be replaced with concrete (encrypted) values during updates. Also, the client locally keeps track of the number cnt_i of values associated to label cnt_i in a table $T_{\mathsf{len}}[\mathsf{cnt}_i] = \mathsf{cnt}_i$.

For each update that inserts the i-th value v_i matching label cnt, the client computes its search token $P_i^{\mathsf{cnt}} \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\mathsf{cnt}, i)$, and sets $\gamma \leftarrow \mathcal{H}_{1\mathsf{C}}(P_i^{\mathsf{cnt}})$ to determine the table T_γ where v_i is stored. Then, she downloads T_γ and the input S_γ^{enc} with which T_γ was setup from the server. After decrypting S_γ^{enc}, she inserts v_i with key P_i^{ℓ} into S_γ, and rebuilds $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$ with the updated input. Then, she sends the updated encrypted input set S_γ^{enc}, table T_γ and seed seed_γ to the server. Note that without rebuilding the table from scratch, we have to change the seed_γ if HTSetup fails during updates, but never during dummy updates, which would allow to distinguish between both operations. As T_i is small, the rebuild process is efficient. Also, we designed CT such that T_γ and seed_γ leak no information about P_i^{ℓ}, though showing this in the context of the security game requires care.

To search label cnt, the client reveals the search tokens P_i^{ℓ} to the server for all $i \in \{1,\dots,T_{\mathsf{len}}[\ell]\}$. These tokens allow the server to recompute the index $\gamma = \mathcal{H}_{1\mathsf{C}}(P_i^{\ell})$ of the hash table T_γ that contains the i-th (encrypted) value associated to cnt. The server then retrieves all encrypted values via $\mathsf{HTLookup}(P_i^{\ell}, T_\gamma, \mathsf{seed}_\gamma)$ and sends the results to the client, who finally decrypts them and filters out false positives.

Remark. When T_γ is accessed during an update, the table T_γ must be rebuilt to ensure forward privacy. For this, we store the content S_γ with which T_γ was built before o the server (in encrypted format). The client then simply rebuilds T_γ via S_γ (after inserting a fresh value if necessary).

Instead, we can store encryptions of value-key pairs $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(v\|k))$ in T_γ instead of encryptions of the values v itself. Then, it is possible to retrieve the input S_γ to HTSetup from T_γ itself.

D.3.1 Security.

Theorem D.1 shows that Dummy1C is secure with respect to definition 13 with standard search and setup leakage and optimal update leakage.

Theorem D.1. *Let $N \geq \lambda$, F be a secure* PRF *mapping to $\{0,1\}^{2\lambda}$ and* Enc *be a secure* IND-CPA *encryption. We model the hash functions $\mathcal{H}_{1\mathsf{C}}$ and $\mathcal{H}_{\mathsf{CT}}$ as random oracles. The scheme* Dummy1C *is correct and supports $\mathsf{poly}(\lambda)(\lambda)$-many dummy updates with leakage $\mathcal{L}^{\mathsf{dummy}} = (\mathcal{L}^{\mathsf{dummy}}_{\mathsf{Stp}}, \mathcal{L}^{\mathsf{dummy}}_{\mathsf{Srch}}, \mathcal{L}^{\mathsf{dummy}}_{\mathsf{Updt}})$, where $\mathcal{L}^{\mathsf{dummy}}_{\mathsf{Stp}}(N, D, M) = N$, $\mathcal{L}^{\mathsf{dummy}}_{\mathsf{Srch}}(\mathsf{cnt}) = (\mathsf{qp}, \mathsf{cnt})$ and $\mathcal{L}^{\mathsf{dummy}}_{\mathsf{Updt}}(\mathsf{op}, \mathsf{cnt}, v) = \perp$.*

[7] This will be convenient for the security proof of Dummy1C.

Algorithm 5 Dummy1C

Dummy1C.KeyGen(1^λ)

1: Sample $\mathsf{K}_{\mathsf{PRF}} \leftarrow \{0,1\}^\lambda$ and encryption key $\mathsf{K}_{\mathsf{Enc}}$
2: **return** $\mathsf{K} = (\mathsf{K}_{\mathsf{PRF}}, \mathsf{K}_{\mathsf{Enc}})$

Dummy1C.Setup(K, N)

1: Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N)) \rceil$
2: Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
3: **for all** $\gamma \in \{0,\ldots,m-1\}$ **do**
4: Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
5: Set $(\mathsf{seed}_\gamma, T_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
6: Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma)$
7: Initialize table $T_{\mathsf{len}}[\ell] = 0$ for all keywords ℓ
8: **return** $\mathsf{EMM} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

Dummy1C.Update($\mathsf{K}, (\ell, v), \mathsf{op}; \mathsf{EMM}$)

Client:

1: Set $\mathsf{cnt} \leftarrow T_{\mathsf{len}}[\ell]$
2: **if** $\mathsf{cnt} = \perp$ **then** set $\mathsf{cnt} \leftarrow -1$
3: Set $P_{\mathsf{cnt}+1}^\ell \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\ell, \mathsf{cnt}+1)$
4: Set $\gamma \leftarrow \mathcal{H}_{1\mathsf{C}}(P_{\mathsf{cnt}+1}^\ell)$
5: **send** γ

Server:

1: Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

1: Decrypt $S_\gamma \leftarrow \mathsf{Dec}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma^{\mathsf{enc}})$
2: Reencrypt all values in S_γ
3: Replace an encrypted zero in S_γ with $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(v), P_{\mathsf{cnt}+1}^\ell)$
4: Run $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
5: Reencrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma)$
6: Increment $T_{\mathsf{len}}[\ell]$
7: **send** $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

1: Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

Dummy1C.Search($\mathsf{K}, \ell; \mathsf{EMM}$)

Client:

1: Set $\mathsf{cnt} \leftarrow T_{\mathsf{len}}[\ell]$
2: **if** $\mathsf{cnt} = \perp$ **then return** $\emptyset$
3: Set $(P_i^\ell) \leftarrow F_{\mathsf{K}_{\mathsf{PRF}}}(\ell, i)$ for $i \in \{1,\ldots,\mathsf{cnt}\}$
4: **send** $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

1: Initialize empty set R
2: **for all** $i \in \{1,\ldots,\mathsf{cnt}\}$ **do**
3: Set $\gamma \leftarrow \mathcal{H}_{1\mathsf{C}}(P_i^\ell)$
4: Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
5: **send** R

Client:

1: Decrypt R to retrieve $\mathsf{MM}(\ell)$

Dummy1C.DummyUpdate($\mathsf{K}; \mathsf{EMM}$)

Client:

1: Set $\gamma \leftarrow \{0,\ldots,m-1\}$
2: **send** γ

Server:

1: Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

1: Decrypt $S_\gamma \leftarrow \mathsf{Dec}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma^{\mathsf{enc}})$
2: Reencrypt all values in S_γ
3: Run $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
4: Reencrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(S_\gamma)$
5: **send** $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

1: Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

While verifying correctness is straight-forward, the proof for semantical dummy security is rather technical. We first sketch security and give a detailed proof below.

First, note that Setup only relies on its input N, and thus trivially leaks no other information than N (or in other words, can be simulated only given N). But to later show that updates and dummy updates are indistinguishable, we also need to make sure that the output of $\mathsf{HTSetup}$, namely T_γ and seed_γ, reveal no information about the keys k_i in the input S_γ.

We first give some intuition for why this is important. Assume that on the contrary, the adversary could learn some k_i in S_γ based on the output of $\mathsf{HTSetup}$. Then, when querying $\mathcal{H}_{\mathsf{CT}}$ on these k_i, *i.e.*, $(\alpha_i,\beta_i) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}_\gamma, k_i)$, the positions $\{\alpha_i,\beta_i\}_i$ always allow for a valid cuckoo hashing assignment. During updates, assume that the keys k_i were replaced with other keys. Then, when querying $\mathcal{H}_{\mathsf{CT}}$ on keys k_i with the a new seed seed'_γ, the new positions $\{(\alpha'_i,\beta'_i) \mid (\alpha'_i,\beta'_i) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}'_\gamma, k_i)\}_i$ are random, as the k_i are not considered anymore during $\mathsf{HTSetup}$ (for deciding whether a seed is sufficient). Thus, they might not allow for a cuckoo assignment anymore with noticeable probability [8]. The above yields a concrete distinguishing attack between a scenario with only dummy updates, and one with only real updates. In summary, when only dummy updates are performed, the outputs of $\mathcal{H}_{\mathsf{CT}}$ always remain valid with regards to cuckoo hashing on input k_i, but is invalid with noticeable probability otherwise.

Thus, we need to show that the outputs $(T_\gamma, \mathsf{seed}_\gamma)$ reveal no information about the keys in S_γ. Intuitively, this follows as $\mathsf{HTSetup}$ only uses these keys to query $\mathcal{H}_{\mathsf{CT}}$, and the output of $\mathcal{H}_{\mathsf{CT}}$ is uniformly random in our model. On a technical level, we show this by simulating the setup with different keys, and reprogramming $\mathcal{H}_{\mathsf{CT}}$ later, such that the distribution of the update and $\mathcal{H}_{\mathsf{CT}}$ output is equally distributed to the real game. This technique also applies after some keys k_i were replaced with some tokens P_i^ℓ during updates. The important consequence is that the simulator can "lazily" reprogram $\mathcal{H}_{\mathsf{CT}}$ for inputs P_i^ℓ until the token was revealed during search, as the adversary is unlikely to query P_i^ℓ beforehand. Also, the simulator can program $\mathcal{H}_{\mathsf{CT}}$ for non-search token inputs k_i arbitrarily, as these are never sent to the adversary, and only queried with negligible probability.

The above allows us to argue that updates leak no information as follows. Observe that the first client output γ looks uniformly random to the server, as $\mathcal{H}_{\mathsf{1C}}$ is modeled as a random oracle. Further, the output of $\mathsf{HTSetup}$ is independent on the used keys (as explained above), and the items stored in the tables T_γ are encrypted. As also S_γ^{enc} is encrypted, the outputs of Update can be simulated with zero leakage (as long as both oracles $\mathcal{H}_{\mathsf{CT}}$ and $\mathcal{H}_{\mathsf{1C}}$ are reprogrammed according to the query leakage during searches). We can proceed exactly the same to simulate dummy updates, and thus both are indistinguishable.

For searches, note that the output of F is pseudorandom. Thus, as in most EMM security proofs, the simulator can simply generate random tokens for not yet queried label-value pairs, and otherwise reuse the tokens from previous searches. This simulation requires the query pattern qp and the number cnt of associated values. Also, for all new tokens, the random oracles $\mathcal{H}_{\mathsf{CT}}$ and $\mathcal{H}_{\mathsf{1C}}$ are reprogrammed to match the expected distribution (which is possible provided qp).

Proof of Theorem D.1. Recall the setup leakage $\mathcal{L}_{\mathsf{Stp}}^{\mathsf{dummy}}(N,D,M) = N$, search leakage $\mathcal{L}_{\mathsf{Srch}}^{\mathsf{dummy}}(\ell) = (\mathsf{qp},\mathsf{cnt})$ and update leakage $\mathcal{L}_{\mathsf{Updt}}^{\mathsf{dummy}}(\mathsf{op},\ell,v) = \perp$ of the leakage function $\mathcal{L}^{\mathsf{dummy}}$. Let $Q_{\mathcal{H}_{\mathsf{CT}}}$ be the number of $\mathcal{H}_{\mathsf{CT}}$-queries, $Q_{\mathcal{H}_{\mathsf{1C}}}$ be the number of $\mathcal{H}_{\mathsf{1C}}$-queries, Q_{Updt} be the number of update queries (including dummy updates) and Q_{Srch} be the number of search queries, performed by the adversary $\mathcal{A}$. The simulator is given in Algorithm 11. We show the claim by providing a sequence of indistinguishable hybrids from the real game to the ideal game.

Hybrid 0. Hybrid 0 is identical to the real game. Note that $\mathcal{H}_{\mathsf{1C}}$ and $\mathcal{H}_{\mathsf{CT}}$ are modeled as programmable random oracles.

Hybrid 1. Hybrid 1 is identical to Hybrid 0, except each search token P_i^ℓ is drawn uniformly at random from $\{0,1\}^{2\lambda}$ instead of being the output of F. Under the PRF security of F, hybrid 1 and hybrid 0 are indistinguishable.

Hybrid 2. Hybrid 2 is identical to Hybrid 1, except instead of generating a random $\gamma \in \{0,\dots,m-1\}$ in a dummy update, first a $P \leftarrow \{0,1\}^{2\lambda}$ is drawn at random, and then $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P)$ is output instead. If P is never queried in random oracle $\mathcal{H}_{\mathsf{1C}}$, then the outputs of Hybrid 2 and Hybrid 1 are identically distributed. Because there are $2^{2\lambda}$ possible values of P, but at most $Q_{\mathcal{H}_{\mathsf{1C}}} + Q_{\mathsf{Updt}}$ evaluations of $\mathcal{H}_{\mathsf{1C}}$, the probability that P was already queried is at most $\frac{Q_{\mathcal{H}_{\mathsf{1C}}}+Q_{\mathsf{Updt}}}{2^{2\lambda}}$. Thus, an adversary can distinguish between Hybrid 1 and Hybrid 2 with advantage at most $\frac{Q_{\mathcal{H}_{\mathsf{1C}}}+Q_{\mathsf{Updt}}}{2^{2\lambda}}$.

Hybrid 3. Hybrid 3 is identical to Hybrid 2, except the tuples S_γ are stored locally for all $\gamma \in \{0,\dots,m-1\}$. During updates, the retrieved encrypted S_γ^{enc} is ignored, and the pair $(\mathsf{Enc}_{\mathsf{K_{Enc}}}(v), P_{\mathsf{cnt}+1}^\ell)$ is inserted into the local S_γ instead which is used as input for $\mathsf{HTSetup}$. The output S_γ^{enc} is an encryption of the local S_γ. Hybrid 2 and Hybrid 3 are distributed identically, as the adversary is honest but curious.

Hybrid 4. Hybrid 4 is identical to Hybrid 3, except all encryptions are replaced with fresh encryptions of 0 (of the appropriate length). Note that $\mathsf{HTSetup}$ behaves independently of the concrete items in S_γ, it just stores them in tables at positions that depend only on the keys. Under $\mathsf{IND\text{-}CPA}$ security of Enc, Hybrid 4 and Hybrid 3 are indistinguishable.

Hybrid 5. Hybrid 5 is given in Algorithm 6 and is identical to Hybrid 4, except during the i-th (dummy or real) update,

[8] This probability is noticeable by the pigeonhole principle. For example if three keys k_1,k_2,k_3 are mapped to the same two bin choices, there is no valid cuckoo assignment. This happens with probability $(\mathsf{cap}+\varepsilon)^{-6}$.

we store the randomly chosen P_i for later. During search, we identify each "real" label with some arbitrary, fresh label ℓ whenever it was first searched for readability, which is the case if sp is 0 for all indices except for the current search. Then, we recompute which cnt tokens $(P_j^{\ell})_{j=1}^{\mathsf{cnt}}$ of the stored $\{P_i\}_i$ match the searched label via the update pattern up, where cnt is the number of matching values, and output these tokens. Hybrid 4 and Hybrid 5 are identically distributed.

Next, we show that the output during updates leak no information about the used keys during HTSetup. This allows us to later argue that we can delay the programming of $\mathcal{H}_{\mathsf{CT}}$ until search.

Hybrid 6. Hybrid 6 is given in Algorithm 7 and is identical to Hybrid 5, except that a flag $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$ is initialized during setup. If during the experiment, the adversary succeeds in setting $\mathsf{flg}_{\mathsf{bad}} = 1$, then output FAIL. Further, during the i-th (real or dummy) update, if some seed seed that was used during HTSetup was already queried in $\mathcal{H}_{\mathsf{CT}}$, set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$. The seeds are chosen uniformly at random during HTSetup, and per update query, at most λ such seeds are chosen. Thus, $\mathsf{flg}_{\mathsf{bad}}$ will be set with probability at most $\frac{(Q_{\mathcal{H}_{\mathsf{CT}}}+\lambda \cdot Q_{\mathsf{Updt}})^2}{2^{2\lambda}}$. As Hybrid 5 and Hybrid 6 are identically distributed, except if FAIL was output, the above probability upper bounds the advantage of $\mathcal{A}$ to distinguish between Hybrid 5 and Hybrid 6.

Hybrid 7. Hybrid 7 is given in Algorithm 8 and is identical to Hybrid 6, except we modify the way the CT hash tables are setup (during setup and updates). That is, we always generate T_γ via $(\mathsf{seed}_\gamma, T_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$ instead of via $\mathsf{HTSetup}(S_\gamma)$. Here, $\mathsf{Remap}(S_\gamma)$ denotes the tuple where the keys in S_γ are replaced with the keys $1,\ldots,n$. Formally, $\mathsf{Remap}((x_i,\tau_i)_{i=1}^{\mathsf{cap}})$ returns the tuple $(x_i,i)_{i=1}^{\mathsf{cap}}$. After the call to HTSetup, we reprogram $\mathcal{H}_{\mathsf{CT}}$ such that its output is distributed as if HTSetup was run with S_γ. In more detail, we set $\mathcal{H}_{\mathsf{CT}}[s,\tau_j] = \mathcal{H}_{\mathsf{CT}}[s,j]$ and $\mathcal{H}_{\mathsf{CT}}[s,j] \leftarrow \bot$, for all seeds s tried during HTSetup and all keys τ_j in S_γ. Note that because there was never a query of the form $\mathcal{H}_{\mathsf{CT}}(s,\cdot)$ before (otherwise $\mathsf{flg}_{\mathsf{bad}}$ was set), the adversary is unaware of the reprogramming. Note that $\mathcal{H}_{\mathsf{CT}}$ is still a valid random oracle, as $\mathcal{H}_{\mathsf{CT}}[s,j]$ is chosen uniformly and independently at random for all j. As HTSetup only uses the keys in S_γ in order to query $\mathcal{H}_{\mathsf{CT}}$, and because we reprogrammed $\mathcal{H}_{\mathsf{CT}}$ accordingly, the distribution of the output of $(\mathsf{seed}_\gamma, T_\gamma)$ is the same in Hybrid 6 and Hybrid 7.

The important consequence of Hybrid 7 is that the output $(T_\gamma, \mathsf{seed}_\gamma)$ of HTSetup reveals no information about the supplied keys. The only part of the experiment that depends on the keys supplied to HTSetup is the state of $\mathcal{H}_{\mathsf{CT}}$.

Hybrid 8. Hybrid 8 is given in Algorithm 9 and is identical to Hybrid 7, except $\mathsf{flg}_{\mathsf{bad}}$ is set if any of the keys k_i in S_γ chosen during setup or some P_i chosen during (real or dummy) updates is queried in $\mathcal{H}_{\mathsf{CT}}$, for the latter conditioned on not yet being revealed during search. As the adversary has information-theoretically no information about these "bad" values (at the time $\mathcal{H}_{\mathsf{CT}}$ is queried) and as they are chosen uniformly and independently at random, the probability that the value k supplied to a query $\mathcal{H}_{\mathsf{CT}}(s,k)$, for any seed s, is equal to a bad value is at most $\frac{(\mathsf{cap}\cdot m + Q_{\mathsf{Updt}})\cdot Q_{\mathcal{H}_{\mathsf{CT}}}}{2^{2\lambda}}$. As Hybrid 7 and Hybrid 8 are identically distributed, except if FAIL was output after some bad value was queried, the above probability upper bounds the advantage of $\mathcal{A}$ to distinguish between Hybrid 7 and Hybrid 8.

Hybrid 9. Hybrid 9 is given in Algorithm 10 and is identical to Hybrid 8, except the insertion of a new pair $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), P_i)$ into S_γ during the i-th update is delayed until the search query that reveals P_i to the adversary. Also, the reprogramming of $\mathcal{H}_{\mathsf{CT}}$ for inputs P_i is done during the revealing search query retroactively. Intuitively, the adversary cannot distinguish Hybrid 9 and Hybrid 8, as $\mathcal{H}_{\mathsf{CT}}$ is never queried on these inputs before the reprogramming is done, and the reprogramming is the only operation that depends on the concrete keys in S_γ.

In more detail, we first store in a table T_{len} the number cnt of values matching label ℓ that were revealed during a search. That is, each search, we store $T_{\mathsf{len}}[\ell] \leftarrow \mathsf{cnt}$, where ℓ is an value for the currently searched label (computed based on the search pattern) and cnt is the number of times ℓ was updated. This allows us to keep track of which P_i was already revealed to the adversary. Further, we (implicitly) keep track of all seeds that were used during the i-th (dummy or real) update in order to reprogram $\mathcal{H}_{\mathsf{CT}}$ later for the revealed P_i during search. Next, we delay the insertion of $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), P_i)$ into S_γ in the i-th update until the first search, where $\mathsf{up}[i] = 1$, *i.e.*the first search that reveals P_i. Note that this does not impact the output of updates, as $(T_\gamma, S_\gamma^{\mathsf{enc}})$ is independent of the keys in S_γ and all items in S_γ are fresh encryptions of zero. But as we reprogram the random oracle $\mathcal{H}_{\mathsf{CT}}$ based on the keys in S_γ after HTSetup during updates in Hybrid 8, the games differ in the output of the random oracles. Notably, all outputs of $\mathcal{H}_{\mathsf{CT}}(\mathsf{seed}, P_i)$ are inconsistent for now. The "correct" outputs are instead stored in $\mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, k_x]$ for some key $k_x \in K_\gamma$ (that should have been replaced during the i-th update). Importantly, these P_i and all such k_x are never queried until revealed during search (otherwise $\mathsf{flg}_{\mathsf{bad}} = 1$). Thus, we simply program $\mathcal{H}_{\mathsf{CT}}(\mathsf{seed}, P_i) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}, k_x)$ during the first search, where $\mathsf{up}[i] = 1$, for all required seeds seed (*i.e.*seeds that were either used during the i-th update or in subsequent updates that rebuilt the same table). Note that via T_{len}, we can identify this search query.

Now, the outputs of $\mathcal{H}_{\mathsf{CT}}(\mathsf{seed}, P_i)$ are distributed equally in Hybrid 8 and Hybrid 9, because these values are never queried before being correctly programmed (otherwise $\mathsf{flg}_{\mathsf{bad}} = 1$). Note that $\mathcal{H}_{\mathsf{CT}}$ is not a valid oracle anymore as its outputs are not randomly distributed for $\mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, k_x]$ (because $\mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, P_i] = \mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, k_x]$). Fortunately, these values are never queried. Thus, Hybrid 8 and Hybrid 9 are identically

distributed.

Hybrid 10. Hybrid 10 is identical to the ideal game with the simulator given in Algorithm 11. Note that real updates and dummy updates are identical in Hybrid 9, as the update specific operations are delayed until the search queries. Hybrid 9 and Hybrid 10 are identically distributed. This concludes the proof.

□

[9] The required values seed are exactly all seeds that were used to reconstruct T_γ during some (dummy or real) update query issued after the definition of P_j. See Appendix D.3.1 for more details.

Algorithm 6 The fifth hybrid. All modifications until here are standard in EMM literature. The modifications are marked in blue boxes.

Dummy1C.Setup(K, N)

Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N))\rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0,\dots,m-1\}$ **do**
  Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$ ▷ Hybrid 3
  $(\mathsf{seed}_\gamma, T_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
  Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$ ▷ Hybrid 4
  Store S_γ locally ▷ Hybrid 3
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{IC}}, \mathcal{H}_{\mathsf{CT}}$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

Dummy1C.Search($\mathsf{K}, \mathsf{qp} = (\mathsf{sp}, \mathsf{up}), \mathsf{cnt}; \mathsf{EDB}$)

Client:

if $\mathsf{cnt} = 0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp ▷ Hybrid 5
Let $\mathcal{J} = (j : \mathsf{up}[j] = 1)$ be the indices of updates on the searched label ▷ Hybrid 5
for all $j \in \mathcal{J}$ in ascending order **do**
  Let i be the index of j in $\mathcal{J}$ ▷ Hybrid 5
  Set $P_i^\ell \leftarrow P_j$ ▷ Hybrid 5
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1,\dots,\mathsf{cnt}\}$ **do**
  Set $\gamma \leftarrow \mathcal{H}_{\mathsf{IC}}(P_i^\ell)$
  Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
send R

Dummy1C.Update($\mathsf{K}, \mathsf{op}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$ ▷ Hybrid 1
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{IC}}(P_i)$ ▷ Hybrid 2
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in local S_γ ▷ Hybrid 3
Replace an item in S_γ inserted during setup with $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), P_i)$ ▷ Hybrid 3
Run $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$ ▷ Hybrid 4
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

Dummy1C.DummyUpdate($\mathsf{K}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$ ▷ Hybrid 1
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{IC}}(P_i)$ ▷ Hybrid 2
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in local S_γ ▷ Hybrid 3
Run $(T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$ ▷ Hybrid 4
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathcal{H}_{\mathsf{IC}}(x)$

if $\mathcal{H}_{\mathsf{IC}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{IC}}[x] \leftarrow \{0,\dots,m-1\}$
return $\mathcal{H}_{\mathsf{IC}}[x]$

$\mathcal{H}_{\mathsf{CT}}(x)$

if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0,\dots,(1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

Algorithm 7 The sixth hybrid. Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$ if chosen seeds during update were already queried in $\mathcal{H}_{\mathsf{CT}}$. Modifications are marked in blue boxes.

$\mathsf{Dummy1C.Setup}(\mathsf{K}, N)$

Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N)) \rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0,\ldots,m-1\}$ **do**
    Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
    $(\mathsf{seed}_\gamma, T_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
    Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
    Store S_γ locally
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{1C}}, \mathcal{H}_{\mathsf{CT}}$
Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

$\mathsf{Dummy1C.Search}(\mathsf{K}, \mathsf{qp} = (\mathsf{sp}, \mathsf{up}), \mathsf{cnt}; \mathsf{EDB})$

Client:

if $\mathsf{cnt} = 0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp
Let $\mathcal{J} = (j : \mathsf{up}[j] = 1)$ be the indices of updates on the searched label
for all $j \in \mathcal{J}$ in ascending order **do**
    Set $P_i^\ell \leftarrow P_j$, where i is the index of j in $\mathcal{J}$
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1,\ldots,\mathsf{cnt}\}$ **do**
    Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i^\ell)$
    Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
send R

$\mathsf{Dummy1C.Update}(\mathsf{K}, \mathsf{op}; \mathsf{EDB})$

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Replace an item in S_γ inserted during setup with $(\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), P_i)$
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s,\cdot] \neq \perp$ **then**
    $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathsf{Dummy1C.DummyUpdate}(\mathsf{K}; \mathsf{EDB})$

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(S_\gamma)$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s,\cdot] \neq \perp$ **then**
    $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathcal{H}_{\mathsf{1C}}(x)$

if $\mathcal{H}_{\mathsf{1C}}[x] = \perp$ **then**
    $\mathcal{H}_{\mathsf{1C}}[x] \leftarrow \{0,\ldots,m-1\}$
return $\mathcal{H}_{\mathsf{1C}}[x]$

$\mathcal{H}_{\mathsf{CT}}(x)$

if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
    $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0,\ldots,(1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

Algorithm 8 The seventh hybrid. Use different keys during HTSetup and reprogram $\mathcal{H}_{\mathsf{CT}}$ afterwards. $\mathsf{Remap}(S_\gamma)$ denotes the function that maps tuples $S_\gamma = (x_i, \tau_i)_{i=1}^{\mathsf{cap}}$ to tuple $(x_i, i)_{i=1}^{\mathsf{cap}}$, *i.e.*, it replaces the keys with $[1, \mathsf{cap}]$.

$\underline{\mathsf{Dummy1C.Setup}(\mathsf{K}, N)}$

Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N)) \rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0, \ldots, m-1\}$ **do**
  Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K_{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
  Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
  for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
    $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
    $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
  Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
  Store S_γ locally
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{1C}}, \mathcal{H}_{\mathsf{CT}}$
Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

$\underline{\mathsf{Dummy1C.Update}(\mathsf{K}, \mathsf{op}; \mathsf{EDB})}$

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Replace an item in S_γ inserted during setup with $(\mathsf{Enc}_{\mathsf{K_{Enc}}}(0), P_i)$
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\underline{\mathcal{H}_{\mathsf{1C}}(x)}$

if $\mathcal{H}_{\mathsf{1C}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{1C}}[x] \leftarrow \{0, \ldots, m-1\}$
return $\mathcal{H}_{\mathsf{1C}}[x]$

$\underline{\mathsf{Dummy1C.Search}(\mathsf{K}, \mathsf{qp} = (\mathsf{sp}, \mathsf{up}), \mathsf{cnt}; \mathsf{EDB})}$

Client:

if $\mathsf{cnt} = 0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp
Let $\mathcal{J} = (j : \mathsf{up}[j] = 1)$ be the indices of updates on the searched label
for all $j \in \mathcal{J}$ in ascending order **do**
  Set $P_i^\ell \leftarrow P_j$, where i is the index of j in $\mathcal{J}$
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1, \ldots, \mathsf{cnt}\}$ **do**
  Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i^\ell)$
  Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
send R

$\underline{\mathsf{Dummy1C.DummyUpdate}(\mathsf{K}; \mathsf{EDB})}$

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\underline{\mathcal{H}_{\mathsf{CT}}(x)}$

if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0, \ldots, (1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

Algorithm 9 The eight hybrid. Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$ if some keys from S_γ are queried to $\mathcal{H}_{\mathsf{CT}}$ before being revealed in Search.

Dummy1C.Setup(K, N)

Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N))\rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0,\dots,m-1\}$ **do**
  Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K_{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
  Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
  for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
    $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
    $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
  Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
  Store S_γ and its keys K_γ locally
Initialize empty set S_P
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{1C}}, \mathcal{H}_{\mathsf{CT}}$
Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

Dummy1C.Update($\mathsf{K}, \mathsf{op}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Replace an item in S_γ inserted during setup with $(\mathsf{Enc}_{\mathsf{K_{Enc}}}(0), P_i)$
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathcal{H}_{\mathsf{1C}}(x)$

if $\mathcal{H}_{\mathsf{1C}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{1C}}[x] \leftarrow \{0,\dots,m-1\}$
return $\mathcal{H}_{\mathsf{1C}}[x]$

Dummy1C.Search($\mathsf{K}, \mathsf{qp} = (\mathsf{sp}, \mathsf{up}), \mathsf{cnt}; \mathsf{EDB}$)

Client:

if $\mathsf{cnt} = 0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp
Let $\mathcal{J} = (j : \mathsf{up}[j] = 1)$ be the indices of updates on the searched label
for all $j \in \mathcal{J}$ in ascending order **do**
  Set $P_i^\ell \leftarrow P_j$, where i is the index of j in $\mathcal{J}$
Add $\{P_i^\ell\}_{i=1}^{\mathsf{cnt}}$ to S_P
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1,\dots,\mathsf{cnt}\}$ **do**
  Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i^\ell)$
  Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
send R

Dummy1C.DummyUpdate($\mathsf{K}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K_{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathcal{H}_{\mathsf{CT}}(x)$

Parse $x = (s, k)$
if $k \in K_\gamma$ for some γ or $k = P_i \notin S_P$ **then**
  Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0,\dots,(1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

Algorithm 10 The ninth hybrid. Delay reprogramming the oracle for search.

Dummy1C.Setup(K, N)

Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N))\rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0,\ldots,m-1\}$ **do**
  Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0), k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
  Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
  for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
    $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
    $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
  Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
  Store S_γ and its keys K_γ locally
Initialize empty set S_P
Store table T_{len} initialized with 0
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{1C}}, \mathcal{H}_{\mathsf{CT}}$
Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}}, T_\gamma, \mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

Dummy1C.Search($\mathsf{K}, \mathsf{qp} = (\mathsf{sp}, \mathsf{up}), \mathsf{cnt}; \mathsf{EDB}$)

Client:

if $\mathsf{cnt} = 0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp
Let $\mathcal{J} = (j : \mathsf{up}[j] = 1)$ be the indices of updates on the searched label
for all $j \in \mathcal{J}$ in ascending order **do**
  Set $P_i^\ell \leftarrow P_j$, where i is the index of j in $\mathcal{J}$
Set $\mathsf{cnt}' \leftarrow T_{\mathsf{len}}[\ell]$
for all $P_i^\ell = P_j$ with $i > \mathsf{cnt}'$ **do**
  Replace some key k_x from S_γ with P_i^ℓ
  Reprogram $\mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, P_i^\ell] \leftarrow \mathcal{H}_{\mathsf{CT}}[\mathsf{seed}, k_x]$ for all required values seed [9]
Set $T_{\mathsf{len}}[\ell] \leftarrow \mathsf{cnt}$
Add $\{P_i^\ell\}_{i=1}^{\mathsf{cnt}}$ to S_P
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1,\ldots,\mathsf{cnt}\}$ **do**
  Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i^\ell)$
  Add $\mathsf{HTLookup}(P_i^\ell, T_\gamma, \mathsf{seed}_\gamma)$ to R
send R

Dummy1C.Update($\mathsf{K}, \mathsf{op}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

Dummy1C.DummyUpdate($\mathsf{K}, \mathsf{op}; \mathsf{EDB}$)

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}} + 1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma, S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}}, T_\gamma, \mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j, \tau_j) \in S_\gamma$ **do**
  $\mathcal{H}_{\mathsf{CT}}[s, \tau_i] = \mathcal{H}_{\mathsf{CT}}[s, j]$
  $\mathcal{H}_{\mathsf{CT}}[s, j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s, \cdot] \neq \perp$ **then**
  $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma, S_\gamma^{\mathsf{enc}}, \mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma, S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathcal{H}_{\mathsf{1C}}(x)$

if $\mathcal{H}_{\mathsf{1C}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{1C}}[x] \leftarrow \{0,\ldots,m-1\}$
return $\mathcal{H}_{\mathsf{1C}}[x]$

$\mathcal{H}_{\mathsf{CT}}(x)$

Parse $x = (s, k)$
if $k \in K_\gamma$ for some γ or $k = P_i \notin S_P$ **then**
  Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
  $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0,\ldots,(1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

Algorithm 11 A description of the simulator.

$\mathsf{Setup}(N = \mathcal{L}_{\mathsf{Stp}}^{\mathsf{dummy}}(N,D,M))$

Sample encryption key $\mathsf{K}_{\mathsf{Enc}}$
Set $m \leftarrow \lceil N/(\delta(\lambda)\log(N))\rceil$
Set $\mathsf{cap} \leftarrow O(\delta(\lambda)\log(N))$
for all $\gamma \in \{0,\ldots,m-1\}$ **do**
    Set $S_\gamma \leftarrow (\mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0),k_i)_{i=1}^{\mathsf{cap}}$ for $k_i \leftarrow \{0,1\}^{2\lambda}$
    Run $(S_{\mathsf{seeds}},T_\gamma,\mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
    for all $s \in S_{\mathsf{seeds}}$ and $(x_j,\tau_j) \in S_\gamma$ **do**
        $\mathcal{H}_{\mathsf{CT}}[s,\tau_i] = \mathcal{H}_{\mathsf{CT}}[s,j]$
        $\mathcal{H}_{\mathsf{CT}}[s,j] = \perp$
    Set $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
    Store S_γ and its keys K_γ locally
Initialize empty set S_P
Store table T_{len} initialized with 0
Initialize counter $\mathsf{cnt}_{\mathsf{updt}} \leftarrow 0$
Initialize empty tables $\mathcal{H}_{\mathsf{1C}},\mathcal{H}_{\mathsf{CT}}$
Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 0$
return output $\mathsf{EDB} = (S_\gamma^{\mathsf{enc}},T_\gamma,\mathsf{seed}_\gamma)_{\gamma=0}^{m-1}$

$(\mathsf{Dummy})\mathsf{Update}(\perp = \mathcal{L}_{\mathsf{Updt}}^{\mathsf{dummy}}(\mathsf{op},\ell,v))$

Client:

Set $i \leftarrow \mathsf{cnt}_{\mathsf{updt}}+1$ and increment $\mathsf{cnt}_{\mathsf{updt}}$
Set $P_i \leftarrow \{0,1\}^{2\lambda}$
Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i)$
send γ

Server:

Send $(T_\gamma,S_\gamma^{\mathsf{enc}})$

Client:

Reencrypt all values in S_γ (stored by simulator)
Save state of $\mathcal{H}'_{\mathsf{CT}} \leftarrow \mathcal{H}_{\mathsf{CT}}$
Run $(S_{\mathsf{seeds}},T_\gamma,\mathsf{seed}_\gamma) \leftarrow \mathsf{HTSetup}(\mathsf{Remap}(S_\gamma))$
for all $s \in S_{\mathsf{seeds}}$ and $(x_j,\tau_j) \in S_\gamma$ **do**
    $\mathcal{H}_{\mathsf{CT}}[s,\tau_i] = \mathcal{H}_{\mathsf{CT}}[s,j]$
    $\mathcal{H}_{\mathsf{CT}}[s,j] = \perp$
if $\exists s \in S_{\mathsf{seeds}} : \mathcal{H}'_{\mathsf{CT}}[s,\cdot] \neq \perp$ **then**
    $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
Encrypt $S_\gamma^{\mathsf{enc}} \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}((0,0)_{i=1}^{\mathsf{cap}})$
send $(T_\gamma,S_\gamma^{\mathsf{enc}},\mathsf{seed}_\gamma)$

Server:

Store updated $T_\gamma,S_\gamma^{\mathsf{enc}}$ and seed_γ

$\mathsf{Search}(((\mathsf{sp},\mathsf{up}),\mathsf{cnt}) = \mathcal{L}_{\mathsf{Srch}}^{\mathsf{dummy}}(\ell))$

Client:

if $\mathsf{cnt}=0$ **then return** $\emptyset$
Recompute value ℓ of the searched label via sp
Let $\mathcal{J} = (j : \mathsf{up}[j]=1)$ be the indices of updates on the searched label
for all $j \in \mathcal{J}$ in ascending order **do**
    Set $P_i^\ell \leftarrow P_j$, where i is the index of j in $\mathcal{J}$
Set $\mathsf{cnt}' \leftarrow T_{\mathsf{len}}[\ell]$
for all $P_i^\ell = P_j$ with $i > \mathsf{cnt}'$ **do**
    Replace some key k_x from S_γ with P_i^ℓ
    Reprogram $\mathcal{H}_{\mathsf{CT}}[\mathsf{seed},P_i^\ell] \leftarrow \mathcal{H}_{\mathsf{CT}}[\mathsf{seed},k_x]$ for all required values seed
Set $T_{\mathsf{len}}[\ell] \leftarrow \mathsf{cnt}$
Add $\{P_i^\ell\}_{i=1}^{\mathsf{cnt}}$ to S_P
send $(P_i^\ell)_{i=1}^{\mathsf{cnt}}$

Server:

Initialize empty set R
for all $i \in \{1,\ldots,\mathsf{cnt}\}$ **do**
    Set $\gamma \leftarrow \mathcal{H}_{\mathsf{1C}}(P_i^\ell)$
    Add $\mathsf{HTLookup}(P_i^\ell,T_\gamma,\mathsf{seed}_\gamma)$ to R
send R

$\mathcal{H}_{\mathsf{1C}}(x)$

if $\mathcal{H}_{\mathsf{1C}}[x] = \perp$ **then**
    $\mathcal{H}_{\mathsf{1C}}[x] \leftarrow \{0,\ldots,m-1\}$
return $\mathcal{H}_{\mathsf{1C}}[x]$

$\mathcal{H}_{\mathsf{CT}}(x)$

Parse $x = (s,k)$
if $k \in K_\gamma$ for some γ or $k = P_i \notin S_P$ **then**
    Set $\mathsf{flg}_{\mathsf{bad}} \leftarrow 1$
if $\mathcal{H}_{\mathsf{CT}}[x] = \perp$ **then**
    $\mathcal{H}_{\mathsf{CT}}[x] \leftarrow \{0,\ldots,(1+\varepsilon)\mathsf{cap}\}^2$
return $\mathcal{H}_{\mathsf{CT}}[x]$

D.3.2 Efficiency.

Dummy1C achieves a constant efficiency overhead for storage, as well as constant communication and computational complexity for searches. For updates, it incurs a $\widetilde{O}(\log N)$ overhead in communication and computational complexity, due to the use of the one-choice process. In Section 5, we will use Dummy1C as a building block that handles full pages of values. In that setting, the previous efficiency features translate to constant storage efficiency, constant page efficiency for searches, and page efficiency $\widetilde{O}(\log N)$ for updates.

Rationale.

Instead of using 1C as the underlying allocation algorithm in Dummy1C, we could also try the same approach with 2C, *i.e.*, using m hash tables $T_0, \ldots, T_{m-1}$, each with a capacity of $\widetilde{O}(\log\log N)$, where $m = N/\widetilde{O}(\log\log N)$. During an update, the i-th value matching label cnt is inserted in the least loaded table amongst T_μ and T_ν for $(\mu,\nu) \leftarrow \mathcal{H}_{1\mathsf{C}}(P_i^\ell)$. During a search on label cnt, both tables T_μ and T_ν are queried on key P_i^ℓ for all $i \in \{1, \ldots, T_{\mathsf{len}}[\ell]\}$.

Indeed, we believe that this approach is fine if during HTSetup, the seed is chosen once at random, and in case of failure, the overflowing items are moved to a stash (see [33]). In that case, the positions $(\alpha_\mu,\beta_\mu) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}_\mu, P_i^\ell)$ (resp. $(\alpha_\nu,\beta_\nu) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}_\nu, P_i^\ell)$) and seed seed_μ (resp. seed_ν) in T_μ (resp. T_ν) are uniformly random. But as there are M tables, we cannot allow for client-side stashes due to efficiency. Therefore, we must repeat with a new seed if allocation fails (as is done in our 1C-based construction). For 2C, the retry procedure induces a specific distribution for the positions $\mathcal{H}_{\mathsf{CT}}(\mathsf{seed}_i, \cdot)$ for tables T_i. Indeed, observe that the set of positions $\{(\alpha_i,\beta_i) \mid (\alpha_i,\beta_i) \leftarrow \mathcal{H}_{\mathsf{CT}}(\mathsf{seed}_i, k_i)\}$ always constitute a valid cuckoo hashing assignment, where k_i are the keys used during setup. If on the other hand some of the keys in the set of positions were not part of the hash table setup input, then it might not constitute a valid cuckoo hashing assignment. Thus, if during a search the server observes that some α_i, β_i are "incompatible" (with respect to cuckoo hashing) with previously observed keys of table T_i, she can guess that one of the accesses in T_i was not a real access.

In summary, as the seeds are conditioned on yielding a valid cuckoo assignment for the keys supplied in setup, the server might infer information about which of the two tables T_μ or T_ν contains a searched value, and which does not. This allows to infer some information on the loads of the tables T_i.

Further, as $\mathsf{poly}(\lambda)$ dummy updates are allowed, some of the pairs of random bins dummy updates might be "incompatible" (with respect to 2C) with the inferred loads. This, in theory, would allow the server to non-trivially distinguish between dummy updates and real updates. While the above sketch does not constitute a concrete attack, we are not aware of an approach to argue that this information cannot be leveraged by an adversary on semantic dummy security.

E Deferred Security Analysis

E.1 Security of our Static EMM

We analyze security of S1C formally (cf. Section 3).

Theorem E.1. *Let $N \geq \lambda$, F be a secure F mapping to $\{0,1\}^{4\lambda+\lceil \log N\rceil}$ and* Enc *be an* IND-CPA*-secure encryption scheme. We model the hash functions $\mathcal{H}_P$ and $\mathcal{H}_{1C}$ as random oracles. The scheme* S1C *is correct and has leakage $\mathcal{L}^{\mathsf{S1C}} = (\mathcal{L}^{\mathsf{S1C}}_{\mathsf{Stp}}, \mathcal{L}^{\mathsf{S1C}}_{\mathsf{Srch}})$, where $\mathcal{L}^{\mathsf{S1C}}_{\mathsf{Stp}}(\mathsf{MM}, N, M) = N$ and $\mathcal{L}^{\mathsf{S1C}}_{\mathsf{Srch}}(\ell) = (\mathsf{sp}, \mathsf{cnt})$.*

Proof. Let $\mathcal{A}$ be a PPT adversary. The simulator Sim proceeds as follows.

Setup. During setup, Sim receives $\mathcal{L}^{\mathsf{S1C}}_{\mathsf{Stp}}(\mathsf{MM}, N, M) = N$. It sets up a simulated table T_{len} by sampling N keys $K_1, \ldots, K_N \leftarrow \{0,1\}^{2\lambda}$ and values $X_1, \ldots, X_N \leftarrow \{0,1\}^{\lceil \log N\rceil}$, and setting $T_{\mathsf{len}}[K_i] \leftarrow X_i$. Also, it simulates T_{full} by sampling $P_i \leftarrow \{0,1\}^{2\lambda}$ and sets $T_{\mathsf{full}}[P_i] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0)$. (Note that 0 represents a dummy value of size $p \cdot |\mathcal{V}|_b$, *i.e.*, an entire page.) Similarly, it simulates $T_{\mathsf{val,i}}$ by setting $T_{\mathsf{val,i}}[P_{i,j}] \leftarrow \mathsf{Enc}_{\mathsf{K}_{\mathsf{Enc}}}(0)$, where $P_{i,j} \leftarrow \{0,1\}^{2\lambda}$ for $i \in [0, m-1]$ and $j \in \mathsf{cap}$. (Note that in this case, 0 represents a dummy value of size $|\mathcal{V}|_b$, *i.e.*, a single value.) It outputs the simulated $\mathsf{EMM} = (T_{\mathsf{len}}, T_{\mathsf{full}}, (T_{\mathsf{val},0}, \ldots, T_{\mathsf{val},m-1}))$.

The simulator stores the sampled keys K_i in a set $\mathcal{K}$, and the tokens P_i and $P_{i,j}$ in sets $\mathcal{P} = \{P_i\}_i$ and $\mathcal{P}_i = \{P_{i,j}\}_j$, respectively, for later.

Search. During a search query, the simulator obtains $\mathcal{L}^{\mathsf{S1C}}_{\mathsf{Srch}}(\ell) = (\mathsf{sp}, \mathsf{cnt})$ and simulates (K, μ, P') as follows. If the label ℓ was searched in the past, it outputs the same value (K, μ, P') as in the prior query.

Otherwise, it chooses a value $P' \leftarrow \{0,1\}^{2\lambda}$ and chooses $K \in \mathcal{K}$ at random, removing K from $\mathcal{K}$ in the process. It then sets $\mu \leftarrow T_{\mathsf{len}}[K] \oplus \mathsf{cnt}$. Finally, it programs $\mathcal{H}_P$ as follows. For $\mathsf{pgcnt} = \lceil \mathsf{cnt}/p \rceil$ and $1 \leq j < \mathsf{pgcnt}$, it programs $\mathcal{H}_P(P', j) \leftarrow P_j$ for some token $P_j \in \mathcal{P}$ and removes P_j from $\mathcal{P}$. Next, it sets $\gamma \leftarrow \mathcal{H}_{1\mathsf{C}}(P', \mathsf{pgcnt})$ and programs $\mathcal{H}_P(P', \mathsf{pgcnt}, j] \leftarrow P_{\gamma,j}$ for $j \in [\mathsf{cnt} \bmod p]$, where $P_{\gamma,j} \in \mathcal{P}_\gamma$ is chosen at random and removed subsequently from the set $\mathcal{P}_\gamma$. Finally, the simulator outputs (K, μ, P').

We show that the real and ideal world are indistinguishable with a standard hybrid argument.

Hybrid 0. Hybrid 0 is identical to the real game. Note that $\mathcal{H}_P$ and $\mathcal{H}_{1\mathsf{C}}$ is modeled as programmable random oracle.

Hybrid 1. Hybrid 1 is identical to Hybrid 0, except the F outputs (K_i, μ_i, P_i') are drawn uniformly at random instead of being the output of F during setup. Under the PRF security of F, hybrid 1 and hybrid 0 are indistinguishable.

Hybrid 2. Hybrid 2 is identical to Hybrid 1, except all ciphertexts are replaced with encryptions of 0 (of appropriate length). under IND-CPA security of Enc, hybrid 2 and hybrid 1 are indistinguishable.

Hybrid 3. Hybrid 3 is identical to Hybrid 2, except the tables T_{full} and $T_{\mathsf{val},0},\dots,T_{\mathsf{val},\mathsf{m}-1}$ are replaced with the simulated tables, and the oracle $\mathcal{H}_P$ is programmed during search as in the simulator. Due to the change in hybrid 2, the values stored in the simulated tables are distributed as in the previous hybrid. Furthermore, observe that the tokens $P_i = \mathcal{H}_P(P',j)$ in search are distributed uniform over the set $\mathcal{P}$ (conditioned on not being chosen yet) as these are the keys for T_{full}. Also, the keys $P_{\mathsf{pgcnt},j} \leftarrow \mathcal{H}_P(P',\mathsf{pgcnt},j)$ are distributed uniform over the remaining keys in $\mathcal{P}_\gamma$, as these are the keys for $T_{\mathsf{val},\gamma}$. Thus, the keys are identically distributed unless either (1) $\mathcal{A}$ queries some P'_i before the random oracle $\mathcal{H}_P$ was reprogrammed in search, (2) there is a collision in $\mathcal{H}_P$ or P'_i, or (3) either set $\mathcal{P}$ or $\mathcal{P}_\gamma$ is empty but a value must be chosen for simulation.

For (1), as P'_i is hidden from $\mathcal{A}$ until search and has high min-entropy, the reprogrammed oracle is distributed as in hybrid 2 in $\mathcal{A}$'s view. For (2), the probability that there are collisions is negligible due to a birthday bound argument. For (3), observe that for $\mathcal{P}$ contains $\lfloor \mathsf{cnt}/p \rfloor$ tokens after setup. Note that the multi-map contains at most $\lfloor \mathsf{cnt}/p \rfloor$ full pages of values. In search, exactly one token is removed for each full page, therefore $\mathcal{P}$ will always be non-empty when a token is retrieved and removed. Similarly, observe that γ is uniform for distinct search queries (except with negligible probability). Therefore, lemma 9 applies and we conclude that $\mathcal{P}_\gamma$ remains non-empty (if a token must be chosen from the set). In more detail, observe that per distinct search query at most p tokens are removed from $\mathcal{P}_\gamma$, as only incomplete pages are stored in $T_{\mathsf{val},\gamma}$. Further, γ is uniform for each distinct query (except with negligible probability as we excluded collisions amongst the tokens). In total, at most N distinct values are removed from all sets $\mathcal{P}_0,\dots,\mathcal{P}_{m-1}$. Per search query, we can interpret the removal of $\mathsf{cnt} \bmod p$ tokens from $\mathcal{P}_\gamma$ as a ball of size $\mathsf{cnt} \bmod p$ that is inserted in bin γ in a 1C process. A bin overflows in the 1C process if and only if we must retrieve a token from an empty set $\mathcal{P}_\gamma$. This happens with negligible probability by lemma 9.

Hybrid 4. Hybrid 4 is identical to Hybrid 3, except the table T_{len} is replaced with the simulated table T_{len} and the values (K_i,μ_i) in the search output are computed as in the simulator. The distribution of both hybrids is identical: The table T_{len} contains uniform keys and entries in both hybrids, the distribution of (K_i,μ_i) in hybrid 3 for a query on a fresh label ℓ is uniform in the set $\mathcal{K}$, and μ_i is distributed such that $T_{\mathsf{len}}[K_i] = \mathsf{cnt}_i \oplus \mu_i$.

Observe that Hybrid 4 is distributed as the ideal game with simulator Sim. The statement follows. □

E.2 Security of our Dynamic EMM

We analyze security of D1C formally (cf. Section 5).

Theorem E.2. *Let $N \geq \lambda$, F be a secure F mapping to $\{0,1\}^{2\lambda}$ and Enc be an IND-CPA-secure encryption scheme. We model the hash functions $\mathcal{H}_P$ and $\mathcal{H}_{1C}$ as random oracles. The scheme S1C is correct and has leakage $\mathcal{L}^{\mathsf{D1C}} = (\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Stp}},\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Srch}},\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Updt}})$, where $\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Stp}}(N,M) = N$, $\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Srch}}(\ell) = (\mathsf{qp},\mathsf{cnt})$ and $\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Updt}}(\ell,v) = \bot$.*

Proof. Let $\mathcal{A}$ be a PPT adversary. The simulator Sim is given access to a simulator Sim_Σ and proceeds as follows.

Setup. On setup, Sim receives $\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Stp}}(N,M) = N$ and runs $\mathsf{EMM}_{\mathsf{full}} \leftarrow \mathsf{Sim}_\Sigma(N)$. Sim initializes an update counter $\mathsf{ctr} = 0$. Further, it sets up empty tables T_γ for $\gamma \in \{0,\dots,m-1\}$ as follows. For table T_γ, it initializes an empty table, and then samples cap keys $k_{\gamma,1},\dots,k_{\gamma,\mathsf{cap}} \leftarrow \{0,1\}^{2\lambda}$, and sets $T_\gamma[k_{\gamma,i}] = \mathsf{Enc}(0\|0)$. It outputs $\mathsf{EMM} \leftarrow (\mathsf{EMM}_{\mathsf{full}},(T_0,\dots,T_{m-1}))$. The simulator stores the sampled keys $k_{\gamma,i}$ in a set $\mathcal{K}_\gamma$ for later.

Search. On search on label ℓ, Sim receives $\mathcal{L}^{\mathsf{D1C}}_{\mathsf{Srch}}(\ell) = (\mathsf{qp},\mathsf{cnt}')$ with $\mathsf{qp} = (\mathsf{sp},\mathsf{up})$. It runs $\mathsf{Sim}_{\mathsf{Sim}}(\mathsf{qp}_\Sigma,\mathsf{pgcnt}_\Sigma)$, where qp_Σ and pgcnt_Σ are chosen as follows.

- We set qp_Σ as the zero vector $(0,\dots,0)$ of length ctr. Note that Sim must set $\mathsf{qp}(j) = 1$ if a page matching ℓ was pushed to Σ. As the pages are pushed in random order, it is possible to recompute a suitable pattern qp_Σ given up: If up indicates that a full page is added or an incomplete page becomes full in epoch', then $\mathsf{qp}(\mathsf{epoch}' \cdot M + j) = 1$, where j is chosen at random not-yet chosen values in $[M]$. (If such a value j was chosen in a prior search query on the same label, as indicated by sp, then j is chosen consistently with the prior query.)
- We set pgcnt_Σ as the number of 1s in qp_Σ.

Then, if the search pattern sp indicates that ℓ was queried previously within the same epoch, then Sim outputs $(P_\ell,\mathsf{cnt},\gamma)$ or nothing as in the prior query.

Else, it sets $\mathsf{cnt} \leftarrow \mathsf{cnt}' \bmod p$ and sets $\mathsf{epoch} \leftarrow \lceil \mathsf{ctr}/M \rceil$. If T_γ was not updated this epoch, then sets $\mathsf{epoch} \leftarrow \mathsf{epoch} - 1$. If $\mathsf{cnt} = 0$, then outputs nothing. Otherwise, samples $P_\ell \leftarrow \{0,1\}^{2\lambda}$ at random. To sample γ, it proceeds different depending on the regime:

- If $N < pM$, on the first search on ℓ, samples $\gamma \leftarrow \{0,\dots,m-1\}$ and outputs γ. Else, it chooses γ as in the prior query.
- If $N \geq pM$, on the first search on ℓ, it samples a fresh value $\gamma \in \{0,\dots,m-1\}$ (that was not yet chosen in a prior search query). Else, it chooses γ as in the prior query.

For $i \in [\mathsf{cnt}]$, it then programs $\mathcal{H}_P(P_\ell,i) = k_i$, where $k_i \in \mathcal{K}_\gamma$ and removes k_i from $\mathcal{K}_\gamma$ in the process. Finally, the simulator outputs $(P_\ell,\mathsf{cnt},\gamma)$.

Update. On updates, Sim simply invokes Sim_Σ to simulate an update query. Also, it sets $\gamma \leftarrow \mathsf{ctr} - 1 \bmod M$ and sends γ if $\gamma < m$. On reception of T_γ, Sim sends back an empty table T'_γ that is simulated as in Sim.Setup. Also, Sim updates $\mathcal{K}_\gamma$ accordingly.

We show that the real and ideal world are indistinguishable with a hybrid argument.

Hybrid 0. Hybrid 0 is identical to the real game. Note that $\mathcal{H}_P$ and $\mathcal{H}_{\mathsf{1C}}$ is modeled as programmable random oracle.

Hybrid 1. Hybrid 1 is identical to Hybrid 0, except the F outputs P are drawn uniformly at random from $\{0,1\}^{2\lambda}$ instead of being the output of F during setup. Under the PRF security of F, hybrid 1 and hybrid 0 are indistinguishable.

Hybrid 2. Hybrid 2 is identical to Hybrid 1, except all ciphertexts are replaced with encryptions of 0 (of appropriate length). under IND-CPA security of Enc, hybrid 2 and hybrid 1 are indistinguishable.

Hybrid 3. Hybrid 3 is identical to Hybrid 2, except the updates on Σ are induced by the simulated update pattern up_Σ for Σ. That is, the updates on Σ are reordered according to up_Σ. As π is a random permutation, both Hybrids are identically distributed.

Hybrid 4. Hybrid 4 is identical to Hybrid 3, except Σ is simulated as specified in Sim. Under EMM security with dummy updates of Σ both hybrids are indistinguishable.

Finally, observe that Hybrid 4 is distributed as the ideal game. □

F Additional In-Memory Experiments

In this section, we provide additional experiments on S1C and D1C run in memory. These experiments use the multi-maps derived from 200K emails. The results can be found in Figure 7.

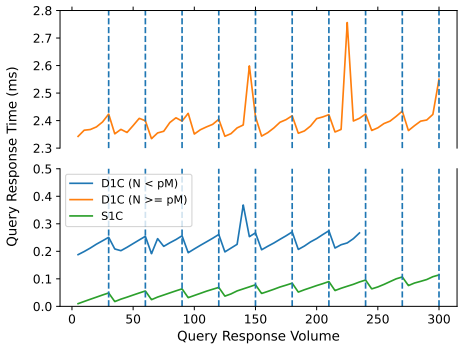


Figure 7 – Search performance of S1C and D1C in memory.